Functions of Time and Space

One thing that you really need to understand "deep down in your bones" is how functions involving spatial dimensions and time behave. So this is a chance to explore this idea and convince yourself of a few basic properties.

Consider a function of one variable, $f(x)$. These are the ones you know and love. All you have to do to turn *any* function $f(x)$ into a traveling wave is to add in a time dependence. Basically all you have to do is replace all the x's in the original $f(x)$ with $(x - t)$'s. No kidding.

Try this. Fire up our very good friend Maple and using the plot command plot any function of x you want. It doesn't matter what function it is. For starters try to choose a function which is not periodic since it makes it easier to see what's happening. If you are at a loss for a function to use try using a Gaussian, e^{-x^2}. Choose plotting limits that give you a reasonable view of the function.

OK, now that you have your function lets "wavify" it. Replace all the x's with $(x - t)$'s. Use the brackets since it'll calculate out properly that way. Now you have two options on how to proceed.

1. **Plot the function as a surface.** To do this use the Maple commands:

   > with(plots);

   > plot3d($f(x,t)$,x=[range of x's], t=[range of t's], style=wireframe);

   where I have used $f(x,t)$ to represent the function you chose above.
   Note: the style parameter just speeds up the plotting. Also you have to wait a while for the plot to be calculated.

   You should now have a surface plotted over a plane defined by x and t coordinate axes. However, this really is not a normal way to look at functions with time dependence. Another more natural (at least to me) way to look at time dependent functions is to watch them change over time like a movie. This is part 2:

2. **Animate the function.** The Maple command you want to use is called animate.
   It calculates 2d plots of functions of (x,t) at successive values of t and then displays the plots as a movie. The calculation takes some time so be patient while it calculates.

   To do this with your function use the following command:

   > animate(($f(x,t)$,x=[range of x's], t=[range of t's]);

   You should get a 2d plot with VCR-like playback buttons on the bottom.
   Press **play** and watch the display. That's the basic idea.

Some things to think about are:

- What happens if I change $(x - t)$ substitutions to $(x - c * t)$ where c is some constant? How does varying c affect the behavior of the function? What do you think c represents?
- Think about which direction the function propagates. Which way would it go if we used $(x + c * t)$? Is this surprising? Convince yourself that the animations Maple is producing are really going in the "right" direction.
- What happens if we put a constant K into the $(x - c * t)$ terms, *i.e.* $(x - c * t + K)$?
- Try adding together two functions that are the same except for using $(x - t)$ for one and $(x + t)$ for the other. You'll want to start them away from each other so use the behaviour you discovered for the K parameter to do this. *E.g.* the following is the form of this sum using $f(x) = x^2$ and "wavifying" it:

  > animate((x–t+4)^2+(x+t–4)^2,x=–10..10,t=0..10);

  Do you understand why you are seeing the behaviour you observe? What would you see for

  > animate((x–t+4)^2–(x+t-4)^2,x=-10..10,t=0..10); ?

NOTE: There is a Web site at `http://grunthos.sci1.ubc.ca/~trevor` with a development of waves similar to what you will encounter in Physics lectures and tutorials this year.