REPRESENTATIONS

In *Art and Science* we pondered the distinction between intuitive knowledge of the particular and analytical knowledge of the abstract. The former governs intimate personal experience — about which, however, nothing further can be said without the latter, since all communication relies upon abstract symbolism of one form or another. We can *feel* without symbols, but we can't *talk*.

Moreover, before two people can communicate they must reach a *consensus* about the symbolic *representation* of reality they will employ in their conversation. This is so obvious that we usually take it for granted, but few experiences are so unsettling as to meet someone whose personal symbolic representation differs drastically from consensual reality.

How was this consensus reached? How arbitrary are symbolic conventions? Do they continue to evolve? They never represent quite the same things for different people; how do we know if there is a reality "out there" to be represented? These are questions that have perplexed philosophers for thousands of years; we are not going to find final answers to them here. But within the oversimplified context of Physics (the social enterprise, the human consensus of paradigmatic conventions, as opposed to *physics*, the actual workings of the universe) we may find some instructive lessons in the interactions between tradition, convention, consensus and analytical logic. This is the focus of the present Chapter.

Each word in a dictionary plays the same role in writing or speech (or in "verbal" thought itself) as the hieroglyphic-looking symbols play in algebraic equations describing the latest ideas in Physics. The big difference is … well, in truth there *isn't* really a big difference. The *small* differences are in compactness and in the degree to which ambiguity depends upon context. Obviously an algebraic symbol like t is rather compact relative to a word composed of several letters, like *time*. This allows storage of more information in less space, which is practical but not always pleasing.

As for ambiguity in context, words are designed to have a great deal of ambiguity until they are placed in sentences, where the context partially dictates which meaning is intended. *But never entirely*. Part of the magic of poetry is its ambiguity; a good poet is offended by the question, "What exactly did you mean by that?" because *all* the possible meanings are intended. Great poetry does not highlight one meaning above all, but rather manipulates the interactions between the several possible interpretations so that each enriches the others and all unite to form a whole greater than the sum of its parts. As a result, no one ever knows for certain what another person is talking about; we merely learn to make good guesses.[1]

In Mathematics, some claim, every symbol must be defined exhaustively and explicitly prior to its use. I will not comment on this claim, but I will pounce on anyone who tries to extend it to Physics. A meticulous physicist will *try* to provide an unambiguous definition of every *unusual* symbol introduced, but there are many symbols that are used so often in Physics to mean a certain thing that they have a well-known "default" meaning as long as they are used in a familiar *context*.

For instance, if $F(t)$ is written on a blackboard in a Physics classroom, it is a good bet that F stands for some *force*, t almost certainly represents *time*, especially when appearing in this form, and the parentheses () *always* denote that F (whatever that is) is *a function of* t (what-

[1] This seems to be holding up progress in Artificial Intelligence (AI) research, where people trying to teach computers to understand "natural language" (human speech) are stymied by the impossibility of reaching a unique logical interpretation of a typical sentence. Methinks they are trying too hard.

ever it may be). This will be discussed further below and in later Chapters. The point is, algebraic notation follows a set of conventions, just like the grammar and syntax of verbal language, that defines the context in which each symbol is to be interpreted and thus provides a large fraction of the meaning of a given expression.

It is tempting to try to distinguish the dictionary from the Physics text by pointing out that every word in the former is defined in terms of the other words, so that the dictionary (plus the grammar of its language) form a perfectly closed, self-reference universe; while all the symbols of Physics refer to entities in the *real* world of *physics*. However, any such distinction is purely æsthetic and has no rigourous basis. Ordinary words are also meant to refer to *things* (*i.e.* personal experiences of reality) or at least to abstract classes of particular experiences. If there is a noteworthy difference, it consists of the potency of the æsthetic commitment to the notion of an external reality. "Natural" language can be applied as effectively in the service of solipsism as materialism, but Physics was designed exclusively to describe a reality independent of human perception, "out there" and immutable, that admits of analytical dissection and conforms to its own hidden laws with absolute consistency. The physicist's task is to discover those laws by ingenuity and patience, and to find ways of expressing them so that other humans can understand them as well.

This may be a big mistake, of course. There may not *be* any external reality; *physics* may be just the consensual symbolic representation of Physics and physicists; or there may not be any physicists other than myself, nor students in my class nor readers of this text, other than in my vivid imagination. But who cares? Solipsism cannot be proven wrong, but it can be proven boring. And since Physics lies at the opposite end of the æsthetic spectrum, no wonder it is so exciting!

3.1 Units and Dimensions

3.1.1 Time and Distance

Two of the most important concepts in Physics are "length" and "time." As is often the case with the most important concepts, neither can be defined except by example — *e.g.* "a meter is this long. . . ." or, "a second lasts from now . . . to now." Both of these "definitions" completely beg the question, if you consider carefully what we are after; they merely define the *units* in which we propose to *measure* distance and time. Except for analogic reinforcements they do nothing at all to explain the "meaning" of the concepts "space" and "time."

Modern science has replaced the standard platinum-iridium reference **meter** (m) stick with the indirect prescription, ". . . the distance travelled by light in empty space during a time of 1/299,792,458 of a second," where a **second** (s) is now defined as the time it takes a certain frequency of the light emitted by cesium atoms to oscillate 9,192,631,770 times.[2] This represents a significant improvement inasmuch as we no longer have to resort to carrying our meter stick to the International Bureau of Weights and Measures in Sèvres, France (or to the U.S. National Bureau of Standards in Boulder, Colorado) to make sure it is the same length as the Standard Meter. We can just build an apparatus to count oscillations of cesium light and mark off how far light goes in 30.663318988 or so oscillations [well, it's easy if you have the right tools. . . .] and make our own meter stick independently, confident that it will come out the same as the ones in France and Colorado, because our atoms are guaranteed to be just like theirs. We can even send signals to neighbors on Tau Ceti IV to tell them what size to make

[2] This is only the latest in a long sequence of redefinitions of the meter. Today's version reflects our recognition of the speed of light as a universal constant. (Here is a trick question for you: if the speed of light *were* different in one time and place from another, how could we tell?)

screwdrivers or crescent wrenches for export to Earth, since there is overwhelming evidence that their atoms also behave exactly like ours. This is quite remarkable, and unprecedented before the discovery of quantum physics; but unfortunately it does not make much difference to the dilemma we face when we try to define "distance." Nature has kindly provided us with an unlimited supply of accurate meter sticks, but it is still just a name we give to something.

To learn the properties of that "something" which we call "distance" requires first that we believe that there is truly a physical entity, with intrinsic properties independent of our perceptions, to which we have given this name. This is extremely difficult to prove. Maybe not impossible, but I'll leave that to the philosophers. For the physicist it is really a matter of æsthetics to enter into conversations with Nature as if there were really a partner in such conversations. In other words, I cannot tell you what "distance" is, but if you will allow me to assume that the word refers to something "real," I can tell you a great deal about its properties, until at some point you feel the partial satisfaction of intimate familiarity where perfect comprehension is denied.

How do we begin to talk about time and space? The concepts are so fundamental to our language that all the words we might use to describe them have them built in! So for the moment we will have to give up and say, "Everyone knows pretty much what we mean by time and distance." This is always where we have to begin. Physics is just like poetry in this respect: you start by accepting a "basis set" of images, without discussion; then you work those images together to build new images, and after a period of refinement you find one day, miraculously, that the new images you have created can be applied to the ideas you began with, giving a new insight into their meaning. This "bootstrap" principle is what makes thinking profitable.

Later on, then, when we have learned to manipulate time and space more critically, we will acquire the means to break down the concepts and take a closer look.

3.1.2 Choice of Units

All choices of units are completely arbitrary and are made strictly for the sake of convenience. If you were a surveyor in 18th-Century England, you would consider the **chain** (66 feet by our standards) an extremely natural unit of length, and the **meter** would seem a completely artificial and useless unit, because people were shorter then and the **yard** (1 yard = 3600/3937 of a meter) was a better approximation to an average person's stride. **Feet** and **hands** were even better length units in those days; and if you hadn't noticed, an **inch** is just about the length of the middle bone in a small person's index finger.

If you couldn't get your hands on a timepiece with a second hand, the utility of **seconds** would seem limited to the (non-coincidental) fact that they are about the same as a resting heartbeat period. **Years** and **days** might seem less arbitrary to us, but we would have trouble convincing our friends on Tau Ceti IV.[3] Remember, our perspective in Physics is universal, and in that perspective all units are arbitrary.

We choose all our measurement conventions for convenience, often with monumental shortsightedness. The decimal number system is a typical example. At least when we realize this we can feel more forgiving of the clumsiness of many established systems of measurement. After all, a totally arbitrary decision is always

[3] This is a recurring problem in science fiction novels: will our descendents on other planets use a "local" definition of years, [months,] days, hours and minutes or try to stick with an Earth calendar despite the fact that it would mean the local sun would come up at a different time every day? Worse yet, how will a far-flung Galactic Empire reckon *dates*, especially considering the conditions imposed by Relativity? [The *Star Trek* solution is, of course, to ignore the laws of physics entirely.]

wrong. (Or always right.)

Physicists are fond of devising “natural units” of measurement; but as always, what is considered “natural” depends upon what is being measured. Atomic physicists are understandably fond of the **Angstrom** (Å), which equals 10^{-10} m, which “just happens” to be roughly the diameter of a hydrogen atom. Astronomers measure distances in **light years**, the distance light travels in a year ($365 \times 24 \times 60 \times 60 \times 2.99 \times 10^8 = 9.43 \times 10^{15}$ m), **astronomical units** (a.u.), which I think have something to do with the Earth’s orbit about the sun, or **parsecs**, which I seem to recall are related to seconds of arc at some distance. [I am not biased or anything....]

Astrophysicists and particle physicists tend to use units in which the velocity of light (a fundamental constant) is dimensionless and has magnitude 1; then times and lengths are both measured in the same units. People who live near New York City have the same habit, oddly enough: if you ask them how far it is from Hartford to Boston, they will usually say, “Oh, about three hours.” This is perfectly sensible insofar as the velocity of turnpike travel in New England is nearly a fundamental constant. In my own work at TRIUMF, I habitually measure distances in **nanoseconds** (billionths of seconds: 1 ns = 10^{-9} s), referring to the distance (29.9 cm) covered in that time by a particle moving at essentially the velocity of light.[4]

In general, physicists like to make *all* fundamental constants dimensionless; this is indeed economical, as it reduces the number of units one must use, but it results in some oddities from the practical point of view. A nuclear physicist is content to measure distances in *inverse pion masses*, but this is not apt to make a tailor very happy.

[4]Inasmuch as a *ns* is a roughly “person-sized” distance unit, it could actually be used rather effectively in place of feet and meters, which would get rid of at least *one* arbitrary unit. Oh well.

3.1.3 Perception Through Models

The upshot of all this is that you can’t trust any units to carry lasting significance; all is vanity. Each and every choice of units represents essentially *a model of what is significant*. What is vitally relevant to one observer may be trivial and ridiculous to another. Lest this seem a depressing appraisal, consider that the same is true of all our means of perception, even including the physical sensing apparatus of our own bodies: our eyes are sensitive to an incredibly tiny fraction of the spectrum of electromagnetic radiation; what we miss is inconceivably vast compared to what we detect. And yet we see a lot, especially under the light of Sol, which at the Earth’s surface happens to peak in just the region of our eyes’ sensitivity. Our eyes are simply a model of what is important locally, and well adapted for the job.

The only understanding you can develop that is independent of units has to do with how dimensions can be combined, juxtaposed, *etc.* — their *relationships* with each other. The notion of a velocity as a ratio of distance to time is a concept which will endure all vagaries of fashion in measurment. This is the sort of concept that we try to pick out of the confusion. This is the sort of understanding for which the physicist searches.

3.2 Number Systems

We have seen that *units* of measurement and indeed the very nature of the *dimensions* of measurement are arbitrary models of what is significant, constructed for the practical convenience of their users. If this causes you some frustration or disappointment, you are not alone; most students of Physics initially approach the subject in hope of finding, at last, some rigor and reliability in an increasingly insubstantial and malleable reality. Sorry.

What most disillusioned Physics students do next is to seek refuge in mathematics. If physical reality is subject to politics, at least the rarefied abstract world of numbers is intrinsically absolute.

Sorry again. Higher mathematics relies on pure logic, to be sure, but the *representation* used to describe all the practically useful examples (*e.g.* "arithmetic") is intrinsically arbitrary, based once again on rather simpleminded models of what is significant in a practical sense. The decimal number system, based as it is upon a number whose only virtue is that most people have that number of fingers and thumbs, is a typical example. If we had only thought to distinguish between fingers and thumbs, using thumbs perhaps for "carrying," we would be counting in *octal* and be able to count up to twenty-four on our hands. Better yet, if we assigned significance to the *order* of which fingers we raised, as well as the *number* of fingers, we could count in *binary* up to 31 on one hand, and up to 1023 using both hands! However, we have already made use of that information for other communication purposes....

Is mathematics then arbitrary? Of course not. We can easily understand the distinction between the *representation* (which is arbitrary) and the *content* (which is not). Ten is still ten, regardless of which number system we use to write it. Much more sophisticated notions can also be expressed in many ways; in fact it may be that we can only achieve a deep understanding of the concept by learning to express it in many alternate "languages."

The same is true of Physics.

3.3 Symbolic Conventions

In Physics we like to use a very compact notation for things we talk about a lot; this is æsthetically mandated by our commitment to making complicated things look [and maybe even *be*] simpler. Ideally we would like to have a single character to represent each paradigmatic "thing" in our lexicon, but in practice we don't have enough characters[5] and we have to re-use some of them in different contexts, just like English!

In principle, any symbol can be used to represent any quantity, or even a non-quantity (like an "*operator*"), as long as it is explicitly and carefully defined. In practice, life is easier with some "default" *conventions* for what various symbols should be assumed to mean *unless otherwise specified*. On the next pages are some that I will be using a lot.[6]

[5] The wider availability of nice typesetting languages like LaTeX, in which this manuscript is being prepared, offers us the opportunity to add new symbols like $\aleph$, ϖ and $\heartsuit$, but this won't change the qualitative situation.

[6] (You will want to refer to these occasionally when trying to guess what I am trying to say with formulae. Don't worry if some are incomprehensible initially; for completeness, the list includes lots of "advanced" stuff.)

Table 3.1 Roman symbols commonly used in Physics

ROMAN LETTERS:

A = an *area*; Ampere(s).	a = *acceleration*; a general constant.
B = magnetic field.	b = a general constant.
C = heat capacity; Coulomb(s).	c = *speed of light*; a gen. constant.
D = a form of the electric field.	d = *differential* operator; *diameter*.
E = *energy*; electric field.	e = 2.71828...; electron's charge.
F = *force*; a general *function*.	f = a fraction; a *function* as in $f(x)$.
G = grav. constant; prefix *Giga-*.	g = *accel. of gravity* at Earth's surface.
H = magnetic field; Hamiltonian op.	h = Planck's constant; a height.
I = electric current.	$i = \sqrt{-1}$; an index (subscript).
J = *Joules*; spin; angular momentum.	j = a common integer index.
K = degrees Kelvin.	k = an integer index; a gen. constant; *kilo-*.
L = *angular momentum*; length.	l = an integer index; a *length*.
M = magnetization; mass; *Mega-*.	m = *metre(s)*; *mass*; an integer index.
N = *Newton(s)*; a large number.	n = a small number; prefix *nano-*.
$\mathcal{O}$ = "order of" symbol as in $\mathcal{O}(\alpha)$.	o = rarely used (looks like a 0).
P = probability; pressure; power.	p = *momentum*; prefix *pico-*.
Q = *electric charge*.	q = *elec. charge*; "canonical coordinate".
R = radius; electrical *resistance*.	r = *radius*.
S = *entropy*; surface area.	s = *second(s)*; distance.
T = *temperature*.	t = *time*.
U = potential energy; internal energy.	u = an abstract variable; a velocity.
V = *Volts*; *volume*; *potential energy*.	v = *velocity*.
W = *work*; weight.	w = a small weight; a width.
X = an abstract function, as $X(x)$.	x = *distance*; any *independent variable*.
Y = an abstract function, as $Y(y)$.	y = an abstract *dependent variable*.
Z = atomic number; $Z(z)$.	z = an abstract *dependent variable*.

Table 3.2 Greek symbols commonly used in Physics

GREEK LETTERS: (Capital Greek letters that look the same as Roman are omitted.)

	α = fine structure constant; an angle.
	$\beta = v/c$; an angle.
Γ = *torque*; a rate.	$\gamma = E/mc^2$; an angle.
Δ = "change in...", as in Δx.	δ = an infinitesimal; same as Δ.
	ϵ = an infinitesimal quantity.
$\mathcal{E}$ = "electromotive force".	ε = an energy.
	ζ = a general parameter.
	η = index of refraction.
Θ = an angle.	θ = an *angle* (most common symbol).
	ι = rarely used (looks like an i).
	κ = arcane version of k.
Λ = a rate; a type of baryon.	λ = *wavelength*; a rate.
	μ = reduced mass; muon; prefix *micro-*.
	ν = *frequency* in cycles/s (Hz); neutrino.
Ξ = a type of baryon.	ξ = a general parameter.
Π = *product* operator.	$\pi = 3.14159\ldots$; pion (a meson).
	ρ = *density* per unit volume; resistivity.
Σ = *summation* operator.	σ = cross section; area density; conductivity.
	τ = a *mean lifetime*; tau lepton.
Υ = an elementary particle.	υ = rarely used (looks like v).
Φ = a *wave function*; an angle.	ϕ = an angle; a wave function.
	χ = susceptibility.
Ψ = a *wave function*.	ψ = a *wave function*.
	ω = *angular frequency* (radians/s).

Table 3.3 Mathematical symbols commonly used in Physics

OPERATORS:

$\rightarrow$ = "...approaches in the limit..." (as in $\Delta t \rightarrow 0$).

∂ = *partial derivative* operator (as in $\frac{\partial F}{\partial x}$).

∇ = *gradient* operator (as in $\nabla\phi = \hat{x}\frac{\partial\phi}{\partial x} + \hat{y}\frac{\partial\phi}{\partial y} + \hat{z}\frac{\partial\phi}{\partial z}$).

$\int$ = *integral* operator as in $\int y(x)dx$

LOGICAL SYMBOLS: (Handy shorthand that I use a lot!)

$\therefore$ = "Therefore..." $\Rightarrow$ = "...implies..." $\equiv$ = "...is *defined* to be..."

$\exists$ = "there exists..." $\ni$ = "...such that..."

/ [a *slash* through any logical symbol] = *negation*; *e.g.* $\not\Rightarrow$ = "...does *not* imply..."

3.4 Functions

Mathematics is often said to be the language of Physics. This is not the whole truth, but it is part of the truth; one ubiquitous characteristic of Physics (the human activity), if not physics (the supposed methodology of nature), is the expression of relationships between measurable quantities in terms of mathematical formulae. The advantages of such notation are that it is concise, precise and "elegant," and that it allows one to calculate quantitative predictions which can be compared with measured experimental results to test the validity of the description.

The nearly-universal image used in such mathematical descriptions of nature is the FUNCTION, an abstract concept symbolized in the form $y(x)$ [read "y of x"] which formally represents *mathematical shorthand* for a *recipé* whereby a value of the "dependent variable" y can be calculated for any given value of the "independent variable" x.

The *explicit* expression of such a *recipé* is always in the form of an *equation*. For instance, the answer to the question, "What is $y(x)$?" may be "$y = 2 + 5x^2 - 3x^3$." This tells us how to get a numerical value of y to "go with" any value of x we might pick. For this reason, in Mathematics (the human activity) it is often formally convenient to think of a function as a *mapping* — *i.e.* a collection of pairs of numbers (x, y) with a concise prescription to tell us how to find the y which goes with each x. In this sense it is also easier to picture the "inverse function" $x(y)$ which tells us how to find a value of x corresponding to a given y. [There is not always a unique answer. Consider $y = x^2$.] On the other hand, whenever we go to *use* an explicit formula for $y(x)$, it is essential to think of it as a *recipé* — *e.g.* for the example described above, "Take the quantity inside the parentheses (whatever it is) and do the following arithmetic on it: first cube whatever-it-is and multiply by 3; save that result and subtract it from the result you get when you multiply 5 by the square of whatever-it-is; finally add 2 to the difference and *voila!* you have the value of y that goes with $x =$ whatever-it-is."

This is most easily understood by working through a few examples, which we will do shortly.

3.4.1 Formulae *vs* Graphs

In Physics we often prefer the image of the GRAPH, because the easiest way to compare *data* with a theoretical function in a holistic manner is to plot both on a common graph. (The right hemisphere is best at holistic perception, so we go right in through the visual cortex.) Fortunately, the issue of whether a graph or an equation is "better" is entirely subjective, because *for every function there is a graph* — although sometimes the interesting features are only obvious when small regions are blown up, or when one or the other variable is plotted on a logarithmic scale, or suchlike.

Nevertheless, this process of translating between left and right hemispheres has far-reaching significance to the practice of Physics. When we draw a *graph*, we cathect the *pattern recognition* skills of our visual cortex, a large region of the brain devoted mainly to forming conceptual models of the "meaning" of visual stimuli arriving through the optic nerve. This is the part that learned to tell the difference between a leaf fluttering in the breeze and the tip of a leopard's tail flicking in anticipation; it performs such pattern recognition without our conscious intervention, and thus falls into the "intuitive" realm of mental functions. It is fantastically powerful, yet not entirely reliable (recall the many sorts of "optical illusions" you have seen).

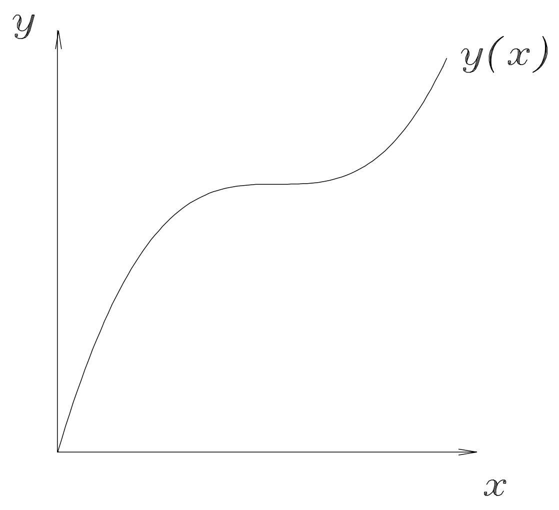

Figure 3.1 A typical graph of $y(x)$ [read "y as a function of x"].

The mere fact that many (not all) physicists like to display their results in graphical form offers a hint of our preferred procedure for hypothesis formation (Karl Popper's *conjectures*). Namely, the data are "massaged" [not the same as "fudged" — massaging is strictly legitimate and all the steps are required to be explained clearly] until they can be plotted on a graph in a form that "speaks for itself" — *i.e.* that excites the strongest pattern-recognition circuit in the part of our visual cortex that we use on science — namely, the straight line. Then the author/speaker can enlist the collaboration of the audience in forming the hypothesis that there is a linear relationship between the two "massaged" variables.

For a simple example, imagine that a force F actually varies inversely with the square of distance r: $F(r) = k/r^2$ with k some appropriate constant. A graph of measured values of F *vs.* r will not be very informative to the eye except to show that, yes, F sure gets smaller fast as r increases. But if the ingenious experimenter discovers by hook or by crook that a plot of F *vs.* $1/r^2$ (or $1/F$ *vs.* r^2 or $\sqrt{F}$ *vs.* $1/r$ or…) comes out looking like a straight line, you can be sure that the data will be presented in that form in the ensuing talk or paper. The rigourous validity of this technique may be questionable, but it works great.