Introduction i

GENERAL NOTES

The experiments which you will perform this semester have been selected with two purposes in mind: to relate physical experience to some of the concepts which you will be learning in class, and to provide an opportunity for you to learn some of the skills and techniques which the physicist uses in exploring the connection between theory and experiment. The laboratory instructors will be available to help you understand the basis of each experiment, but the responsibility for preparation lies with you. You will find that the laboratory experience is most intellectually rewarding, as well as enjoyable, when you arrive prepared for the labs and perform them with care.

Some of the experiments are not directly related to the topics discussed in class, but are intended to acquaint you with the methodology of scientific inquiry. This semester we will particularly emphasize the theory of errors, their origins, and how they influence the results of an experiment. While at first some of the exercises may seem trivial, it cannot be stressed too strongly that the skills associated with analyzing data properly form the backbone of scientific investigation. The outcome of an experiment is only as valid as the method used in obtaining the result.

In the first laboratory exercise you will examine the errors associated with a series of measurements. Before proceeding, you are first asked to consider the simple concept of significant figures and the skill of drawing useful graphs. The essentials are discussed in the next few pages.

i.1 Significant Figures

Suppose that during the process of an experiment you are confronted with the task of determining the circumference of a pulley. You measure the diameter of the pulley with a good ruler whose smallest division is 1/10 inch and record the diameter as 4.15 inches. Taking $\pi = 3.1416$ and multiplying out 3.1416×4.15, you write down: Circumference equals 13.037640 inches. Arithmetically the result is correct. Physically it is misleading. Confronted with the ruler, you would readily admit that you could not estimate with certainty to one-hundredth of an inch, and that the diameter of the pulley might well be 4.14 inches instead of 4.15 inches. If, however, the diameter of the pulley is taken as 4.14 inches, the circumference would come out according to this calculation as 13.006229 inches. Comparing this result with the former, you find that they differ in the fourth figure.

Which answer is correct? You really do not know, and in fact using the apparatus mentioned no one would be willing to say. Another observer might estimate the diameter to be 4.16 inches and a third "circumference" would be obtained. Because of this uncertainty the physicist says that all the answers are wrong since they *explicitly express* an accuracy in the determination of the circumference which is not warranted by the precision in the measurement of the diameter. The physicist says that some of the numbers written down are significant figures and that the rest are meaningless.

How, then, are we to determine the number of significant figures in a result? For use in the analysis of the results of carefully performed experiments there are statistical methods available which allow a prediction to be made of the precision. One such system is outlined in Experiment 1. We shall not use such a system here but content ourselves instead with a less exact method. Returning to our example, we assume that after a number of trial measurements we would be willing to state that the diameter of the pulley is 4.15 ± 0.01 inches (to be read "4.15 plus or minus 0.01 inches"). In other words we are willing to state that the measured diameter is correct to about $\pm 0.2\%$. As will be shown later, the determination of the circumference is then only accurate to $\pm 0.2\%$ and should be written as 13.04 ± 0.03 inches.[1]

[1] Another convention, not so universally respected, is to always place a leading zero before the decimal point in ex-

The experimenter should recognize that *the accuracy of the result is independent of the position of the decimal point in the result.* Changing the units in which the result is expressed can in no way alter the accuracy of the result. The term "significant figures" is often used to give this idea: a result accurate to four significant figures would be one in which the observer relies on the first four figures counting always from the first number at the left, other than zero. As an illustration, if a certain distance is known to be 15.1 meters, accurate to about 1 percent — that is, to three significant figures — then the same result would be 0.0151 kilometers or 1.51×10^4 millimeters, using as before three significant figures.

A frequent misunderstanding arises in the case where the last significant figure is zero. A measurement made with a centimeter rule of the diameter of a cylinder might be 7.10 cm. Here the 0 is a significant figure; the observer could have seen if the measurement had been 7.09 or 7.11. A record of 7.1 cm would indicate that the observer had failed to read as closely as possible, but on the other hand a record of 7.10000 cm would not truthfully represent the observation, since it could not have been possible to observe with the ruler what figures occupied the fourth, fifth and sixth places.

In arithmetical computation, insignificant figures may be dropped at each stage of the computation. The computation of the volume of the cylinder whose dimensions are given below will serve as an illustration.

$$d = 7.138 \pm 0.008 \qquad i.e. \ \pm 1/9\%$$

$$h = 8.291 \pm 0.004 \qquad i.e. \ \pm 1/20\%$$

$$V = \pi r^2 h = \frac{\pi d^2 h}{4}$$

The number 7.138 is accurate to 1/9 percent, therefore every time this number is used as a factor it may introduce an error of 1/9 percent, or the fourth figure will be uncertain. As a consequence, we may omit all figures beyond the fourth which come in as a result of multiplication. Thus we continue the calculation:[2]

$$d^2 = (7.138) \times (7.138) = 50.95 \quad (\textit{not } 50.951044!)$$

$$d^2 h = (50.95) \times (8.291) = 422.4$$

$$V = \frac{\pi}{4} d^2 h = \frac{3.1416}{4} \times 422.4 = 331.7 \ cm^3$$

The procedure for assigning uncertainty to the result will be considered in Experiment 1.

i.1.1 Cautions

Never forget that *significant figures* conventions were invented to prevent misunderstanding and wasted effort, not to establish rigid rules. There *are* circumstances in which common sense dictates violation of the conventions. For instance, suppose a runner finishes a 10 *km* race in 36 minutes, 17.31 seconds; it is reasonable to assume that the distance has actually been measured to the nearest few meters, so if it were expressed in the "sig fig" convention it would be 10,000 *m*. But it usually isn't. Would you then feel obliged to ignore all but the first two significant figures of the mean speed calculated by dividing the distance by the time? Of course not. You are expected to use your judgement in applying significant figures.

Another situation in which significant figures can be misleading is the lengthy calculation involving many combinations of measured numbers with various uncertainties. In such cases there is always the danger that "round-off errors" can accumulate as one proceeds through the steps of the calculation, yielding a final result that differs by more than the last digit from the one made ignoring the significant-figures convention. Therefore it is not considered bad form to retain one *extra* significant figure in all intermediate calculations and save the strict convention for the final result. If you are making calculations in an electronic calculator, of course, there is no reason to truncate *any* of the intermediate results unless you are transcribing them to paper to show your work.

Finally, the significant figures convention (like "52.11") is just a very compact (and slightly crude) means of expressing uncertainty; if you want to say exactly what you mean, just use the explicit notation "52.113 ± 0.006" or the shorthand version "52.113(6)" where the number in parentheses is the uncertainty *in the last digit*. Such explicit notation is always used in describing the results of lengthy and/or expensive experiments like those at high energy particle accelerators.[3] The general rule is,

pressing a number less than one. For instance, we try to avoid writing ".1" rather than "0.1" because the leading decimal point can too easily be overlooked in a cursory glance.

[2] Several other conventions are implicit here: first, *units* are suppressed until the end. This is a time-saving convention which sometimes backfires — in more dimensionally elaborate calculations it is often worth carrying along the units at every step to ensure that no step is "dimensionally absurd" and to aid in locating errors when the final answer doesn't make sense. Second, *integers* (like 4) are exempt from the significant figures convention, for obvious reasons. We could write 4.000 instead, but that would be silly.

[3] In fact, most of the hard work in analyzing the data from elementary particle physics experiments goes into meticulous estimation of uncertainties. You will often see results expressed in the form 2.19732 ± 0.00015 where even the *uncertainty* is expressed to several significant figures!

use common sense and *make sure your notation conveys clearly what you know and what you don't know*.

i.2 Graph Plotting

Since you will be required to express many of your results in the form of graphs, the following suggestions will be of use.

A. The general idea is the same as that used in graphical methods in algebra, with the same conventions in regard to signs and location of points. If the values of the two quantities to be plotted are not already arranged in columns, time will be saved by so arranging them before attempting to plot points on the graph.

B. A scale of units for each axis should in general be chosen so that the final curve will practically (but not necessarily) fill the page, though the unit chosen must be one which can readily be applied. Neglect of this caution causes much waste of time and also defeats the purpose of the graph.

A common difficulty in this respect arises from an attempt to use a scale such as "one square for each three units" on decimal co-ordinate graph paper. A convenient rule for decimal co-ordinate paper is to let one square equal one, two, or five units or any of these three numbers multiplied by a power of ten (*e.g.* 0.1, 0.2, 0.5, or 10, 20, 50).

C. The prominent divisions should be plainly marked on the two axes. Do not mark every square — too many numbers on the axes generally make the graph confusing. The values of the individual readings should not appear on the axes.

D. Each axis should bear a label which includes the unit used.

E. The separate points should be accurately located and marked in such a way as to be distinguishable after the curve is drawn. An encircled dot is especially convenient for locating points. When it is important to record the uncertainties of each plotted point, horizontal and/or vertical bars are passed through the point, the lengths of which represent the uncertainty of the point with respect to the x and/or y co-ordinates.

F. If several curves are drawn on the same sheet, it is best to represent them in different ways, *e.g.* broken and solid lines, and to mark the points of each curve by different symbols.

G. If there is a theory concerning the functional relationship of the two variables plotted, then a curve corresponding to that hypothesis should be drawn through the experimental points. In many experiments the theory is not well known in advance and the only hypothesis that can be made is that it will yield a smooth curve. *The curve should be drawn through the mean of the various points.* The curve need not pass through the first and last points. Instead, each point should be considered as accurate as any other point (unless there are experimental reasons for some points being less accurate than others), so that the curve is drawn with about as many points above the curve as are below it with the 'aboves' and 'belows' distributed at random along the curve (*i.e.* not all points above the curve at one end and below at the other end).

H. Each graph should bear a title telling briefly what the curve represents.

As an example, suppose that we wish to plot the curve showing the relation between the observed position of an indicator on the end of a spring and the force pulling the spring, and suppose that we have made the following observations:

Indicator Position (centimeters)	**Force on Spring (N)**
3.3 ± 0.2	5.00
5.2 ± 0.3	15.00
7.8 ± 0.4	25.00
10.0 ± 0.5	35.00
12.5 ± 0.4	45.00

Which shall we plot along the "horizontal" x-axis and which along the "vertical" y-axis?[4] A general rule is to plot the *independent* variable along the x-axis (also known as the *ordinate*) and the *dependent* variable along

[4] The terms "horizontal" and "vertical" are of course not literal, but based on yet another perceptual convention about the orientation of the two-dimensional surface on which the graph is being drawn; all this may become considerably more confusing when 3D "virtual reality" computer interfaces become more widely used.

the y-axis (also known as the *abcissa*). In other words, if the position of the indicator on the spring is to be expressed in terms of the force on the spring, then plot the force on the x-axis and the indicator position on the y-axis. The scale units are chosen with consideration both for convenience and for the size of the diagram.

We now make the assumption that the relation between the force F applied to the spring and the extension s of the spring is given by HOOKE'S LAW,

$$F = k(s - s_\circ)$$

written to include $s_\circ$, the position of the indicator when $F = 0$.

We therefore expect a linear relation between F and s, so we draw the best straight line through the points.[5]

If we now wish to determine the spring constant, k, we can do so easily from the slope of the line. The slope is obtained by taking two points on the straight line *(NOT two experimental points, because the straight line represents the best fit to the experimental data)*, say (x_1, y_1) and (x_2, y_2), and forming the quotient:

$$\frac{(y_2 - y_1)}{(x_2 - x_1)} \quad = \quad \text{the slope.}$$

As a rule, it is convenient to choose points such that the difference $(x_2 - x_1)$ comes out to be a simple number; in this way, the process of division is simplified. In this example, since F is plotted on the x-axis and s on the y-axis,

$$\text{slope} \quad = \quad \frac{(s_2 - s_1)}{(F_2 - F_1)}$$

where (F_1, s_1) and (F_2, s_2) are any two points on the best fit line and so

$$F_1 = k(s_1 - s_\circ) \qquad\qquad F_2 = k(s_2 - s_\circ).$$

Subtracting the first of the above equations from the second,

$$(F_2 - F_1) = k(s_2 - s_1)$$

so that

$$\text{slope} \quad = \quad \frac{1}{k}.$$

The slope in the example shown on the following page is 0.233 cm/N. Thus

$$k = \frac{1}{\text{slope}} = \frac{1}{0.233} = 4.29 \text{ N/cm}.$$

Note that since we used the slope of the best-fit line rather than a single point to calculate k, we did not need to find $s_\circ$. However, this calculation is only as good as our ability to draw a straight line through the mean of the points. There are criteria, such as the "least squares fit," which make it possible to find the curve which best fits a set of points. If such a method had been applied, the use of the slope would certainly yield a better estimate than a single point or pair of points. Usually we will not use such techniques and will fit curves to data visually. Such a fit is easily made with the aid of a clear flat plastic ruler.

[5] Of course, if we stretch the spring too far we will discover deviations from this ideal behavior. Most physical "laws" are applicable only within certain limits with which the user is expected to be familiar. This expectation is frequently frustrated....

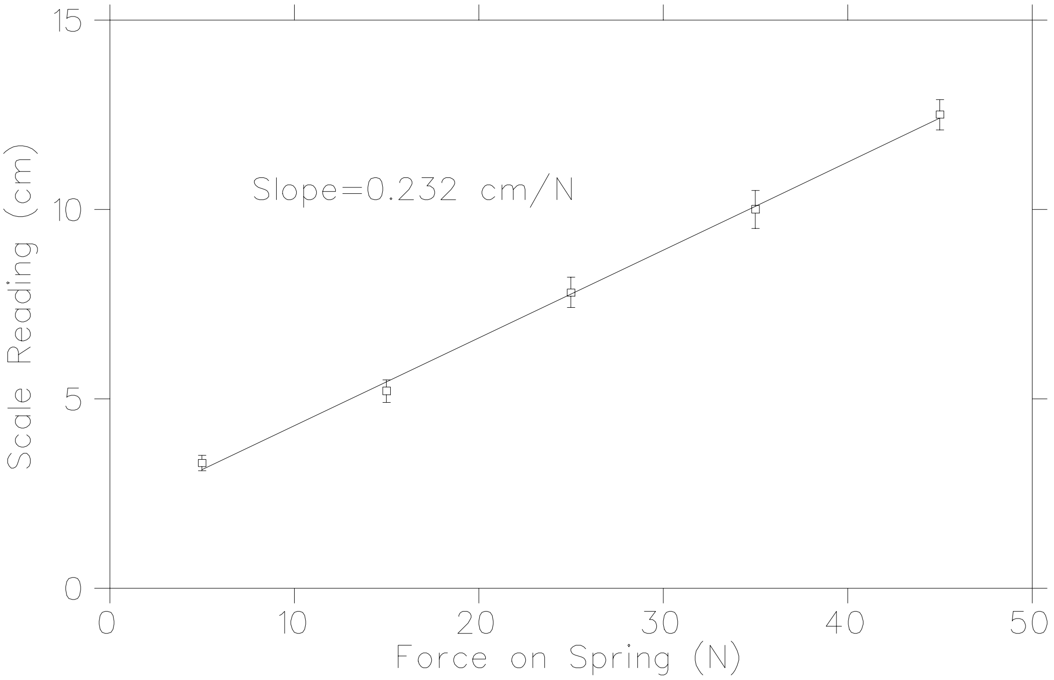

Figure i.1 Hooke's Law: Position of Indicator versus Spring Force