Experiment 10

MESON DECAY:

Analysis of Bubble Chamber Events

Object: To measure the approximate masses of the pion (π meson) and muon (μ lepton) from analysis of digitized photographs of the bubble chamber in which they decayed.

When nuclei are bombarded by high energy particles which are found as cosmic rays or are produced by large accelerators, a variety of unstable "elementary" particles may be produced. Most of the experiments performed in the past twenty years with beams from the growing number of cyclotrons and synchrotrons have involved the discovery of new particles and the determination of their properties, *e.g.* mass, charge, mean life until decay, decay products, interaction with other particles, *etc.*

Visual Detection Techniques

Electronic detectors, such as Geiger counters or scintillation counters, are particularly useful for recording large fluxes of charged particles or for accurate timing measurements. Where more accurate spatial measurements are needed, however, "visual" detection techniques, such as photographic emulsions, cloud chambers, bubble chambers, or spark chambers may be used.[1] All of the visual devices record the trail of ionization left in the wake of a charged particle. In a bubble chamber,

[1] **"Electronic bubble chambers"** — In recent years, a combination of more complex "events" and higher event *rates* has rendered visual analysis of photographic records impractical for many types of high energy elementary particle physics experiments. Instead, one employs position-sensitive detectors such as *wire chambers* (for the invention of which Charpak recently won a Nobel prize) which encode the positions of charged particles as they cross the chamber and transfer those positions to the input buffer of a fast computer. A typical experiment involves many such chambers reporting hundreds of coordinates of particle trajectories for each "event" (where one or two incoming particles collide and any number of particles may exit, all within a few billionths of a second). The computer can transfer thousands of such "events" to a recording medium such as magnetic tape, from which they can later be read and analyzed by other computers. Thus the process of data reduction and analysis is far faster and less subjective than in the days of bubble chambers when human "scanners" were employed to visually examine millions of photographs. Nevertheless, certain types of events are still so rare (and background-free) that bubble chambers are still used in their measurement.

small bubbles which form along the trails of ionized molecules are illuminated and stereoscopically photographed milliseconds after the passage of the particle. The detailed trajectory of the particle can be examined in the photograph. If the chamber was in a strong magnetic field, the momentum of the particle can be determined from the curvature of its trajectory in the plane normal to the field. If the particle comes to rest in the liquid, its kinetic energy can be determined from its range. Since slower particles ionize more heavily, an estimate of the velocity can be made from the density of bubbles along the track.

Relativistic Kinematics

For particles traveling near the speed of light c, the relations among rest mass m_0, velocity $v = \beta c$, time dilation factor $\gamma = (1-\beta^2)^{-1/2}$, relativistic effective mass $m = \gamma m_0$, momentum $p = mv$, kinetic energy T, and total energy $E = mc^2$ must be derived from the special theory of relativity.

The most useful of the derived results for this experiment are

$$E^2 = (pc)^2 + \left(m_0c^2\right)^2 , \tag{1}$$

$$E = T + \left(m_0c^2\right) , \tag{2}$$

$$(pc)^2 = T^2 + 2T\left(m_0c^2\right) \tag{3}$$

$$\text{and} \quad \beta \equiv \frac{v}{c} = \frac{pc}{E} . \tag{4}$$

Note that E, T, pc, and m_0c^2 have dimensions of energy; it is customary in particle physics to express each of them in units of MeV (10^6 electron volts) where one electron volt (eV) equals 1.602×10^{-19} joule (J).

10.1 $\pi - \mu - e$ Decays

Introduction

The π^+ meson (pion) decays, with a mean lifetime of 26.03×10^{-9} s, into a μ^+ lepton and a muon neutrino:

$$\pi^+ \to \mu^+ + \nu_\mu \tag{5}$$

The emission of a *single* neutrino (a neutral particle with zero mass) is deduced from the fact that a constant value of the muon's momentum is observed in all decays of pions at rest. (You should be able to see why this conclusion is justified.)

The μ^+ lepton (muon) then decays, with a mean lifetime of 2.197×10^{-6} s, into a positron (positive electron) and a neutrino - antineutrino pair:

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \tag{6}$$

The emission of *two* massless particles (an electron neutrino and a muon antineutrino) is deduced from the fact that the positrons are emitted with a continuous spectrum of energies from decays of muons at rest, indicating that energy and momentum are shared among 3 decay products.[2]

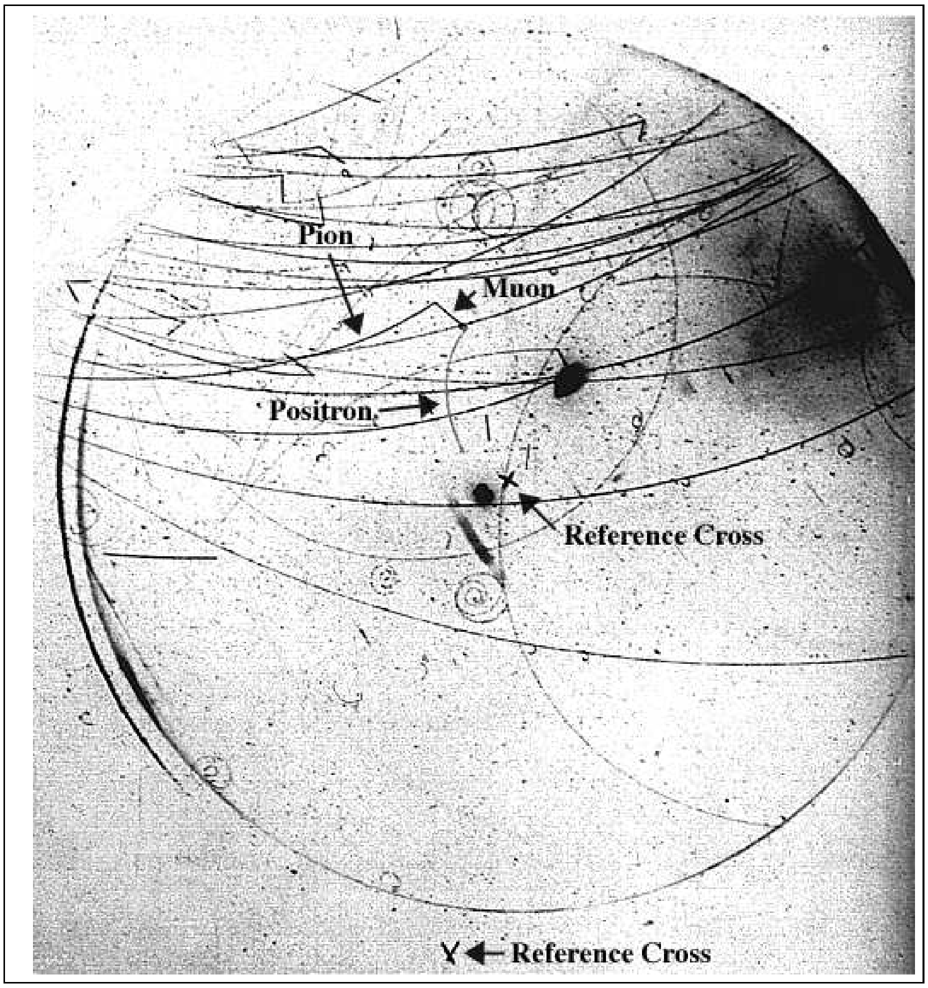


Figure 10.1 — Photograph of a bubble chamber showing pion decays.

Figure 10.1 shows a typical bubble chamber photograph. The pions enter from the upper left, and those that come to rest in the chamber can be seen to decay into muons. The muons leave short,

[2]The fact that there are two (actually three) *types* of neutrinos was deduced from the inability of neutrinos originating from pion decay (5) to produce electrons in high energy scattering events; it is now believed that there are three (and only three) types of *leptons*: the *electron*, the *muon* (often casually referred to as a "heavy electron") and the *much* heavier *tauon*. Each lepton has its own associated neutrino. All you really need to know for this experiment is that there are two massless particles and one very light one (the positron) emitted in muon decay (6).

dense tracks, and then themselves decay into (positive) electrons, which leave longer, lighter tracks. The two reference crosses are used to calibrate measurements of the particle tracks.

10.1.1 Measurement of m_{0_μ}

For massless particles like neutrinos, $E = pc$; the rest mass of the muon (approximately one hundred MeV) is *much* larger than that of the positron (0.511 MeV), so that $E \approx pc$ for the e^+ as well, and $m_{0_\mu} c^2$ goes almost entirely into the kinetic energy of the decay products:

$$E_\mu \;=\; m_{0_\mu} c^2 \;=\; E_e + E_{\nu_e} + E_{\bar{\nu}_\mu} \;\simeq\; p_e c + p_{\nu_e} c + p_{\bar{\nu}_\mu} c \tag{7}$$

The three "light" decay products are thus on an almost equal footing, and if no special law were acting in the reaction, each might be expected to receive, *on average*, 1/3 of the total energy. (The interaction among the decay products is "weak", and detailed experimental results indicate that this expectation is valid to a good approximation.) In any one decay event, the visible positron track can receive any energy from zero up to $E_\mu/2$ [*Prove this!*], but averaged over a large number of events one gets[3]

$$\langle p_e c \rangle \approx \langle p_{\nu_e} c \rangle \approx \langle p_{\bar{\nu}_\mu} c \rangle \approx \frac{E_\mu}{3} = \frac{m_{0_\mu} c^2}{3} \tag{8}$$

Equation (8) serves as the basis for the measurement of m_{0_μ} in this experiment.

10.1.2 Measurement of m_{0_π}

In the decay (5) of a pion at rest, conservation of total energy yields

$$m_{0_\pi} c^2 = \sqrt{(p_\mu c)^2 + \left(m_{0_\mu} c^2\right)^2} + p_{\nu_\mu} c \tag{9}$$

But, by conservation of momentum in the two-body decay,

$$p_\mu = p_{\nu_\mu} \; . \tag{10}$$

Therefore

$$m_{0_\pi} c^2 = \sqrt{(p_\mu c)^2 + \left(m_{0_\mu} c^2\right)^2} + p_\mu c \tag{11}$$

Appendix I shows how a knowledge of the magnitude B of the magnetic field and the radius of curvature R of the electron track permit one to calculate p_e, from which m_{0_μ} may be found. The

[3] The notation "$\langle X \rangle$" means "the average value of X".

unique value of T_μ (the kinetic energy of the decay muon) can be determined by equations (22) and (24) from the measurement of $\langle L_\perp \rangle$, which is the average of the projected range of the muon tracks from a large number of decays.

From T_μ and a knowledge of m_{0_μ}, a value for p_μ can be obtained; substituting p_μ and m_{0_μ} in Eq. (11) yields a value for m_{0_π}.

10.2 PROCEDURE

Experimental Arrangement

The bubble chamber photographs to be scanned were taken using the 385 MeV proton synchrocyclotron at Columbia's Nevis Laboratory. Pions were produced by collisions of accelerated protons with a copper target inside the vacuum chamber of the cyclotron. The pion beam emerged through a thin window, passed through a hole in the surrounding shielding, and was allowed to enter a liquid hydrogen bubble chamber.

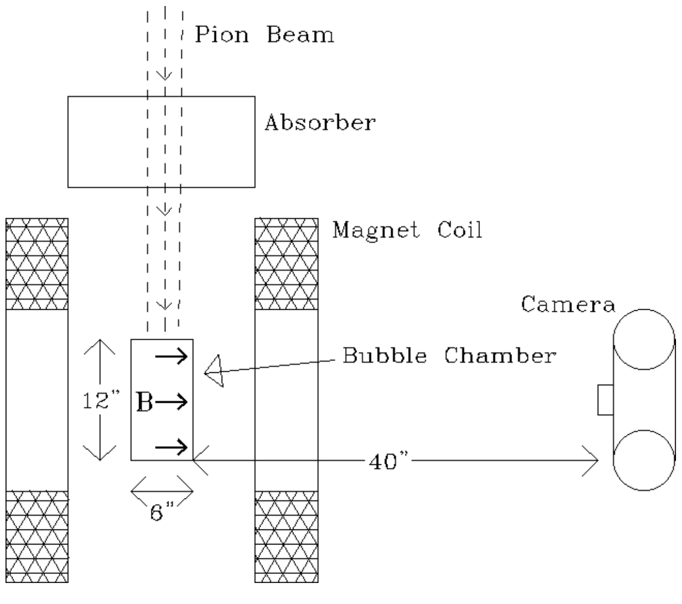


Figure 10.2 — Arrangement of bubble chamber and auxiliary equipment.

The experimental arrangement is indicated in Fig. 10.2. An absorber is placed in the pion beam, so that a sizable fraction of the incident pions come to rest in the liquid hydrogen. The emerging muon travels a short distance before coming to rest, and the final positron makes a long, lightly ionizing track, passing through the hydrogen until it leaves the chamber. If the positron leaves through a side of the chamber which is parallel to the picture plane, the track will appear to stop at an arbitrary point in the projected view.

10.2.1 Measurements

You are to analyze the digitized photographs from the bubble chamber on a computer using an X-terminal with pre-installed software that was developed to meet the needs of this type of analysis. For a large number of identifiable $\pi - \mu - e$ events, you are to measure:

A. The average radius of curvature of the positrons, from which the average positron momentum and, finally, the mass of the muon are derived.

B. The length of the muon track, from which the muon momentum and, finally, the mass of the pion are derived.

The software includes commands for calibration and for measuring the straight-line length and radius of curvature of tracks. See **Appendix II** for instructions on using the software. In order to convert distances on the photographs into actual distances in the bubble chamber, four marks were scratched into the glass on the front of the chamber. Three of these marks are around the outside rim and form an equilateral triangle 22.4 cm on a side, but the mark on the upper-right is obscured in many of the photographs. Instead you should calibrate using the two Reference Crosses indicated in Fig. 10.1, which are 13.8 cm apart. An additional correction factor involves the fact that while the front of the bubble chamber is 40 cm from the camera, the average track in the chamber is 43 cm from the camera; therefore, the calibration reference distance must be multiplied by 43/40. The software does this calculation automatically, so that when you click on the two reference points it assigns them a distance of 14.8 cm. Hence all of the measurements you obtain with the software are already fully corrected.

In using the average value of $\sin\theta$ (to calculate L from $\langle L_\perp \rangle$) for the average of the large number of events measured, it is assumed that there is no scanning bias — *i.e.*, that you are as likely to identify and measure one event as another. Although this goal is unlikely to be achieved, try not to favor events in the picture plane by failing to measure events in which the projection of the track is short.

10.2.2 Calculations

Constants: $B = 8800$ gauss
ρ of liquid hydrogen $= 0.07$ gm/cm^3

Find the average radius of curvature of the positrons. From this, calculate the average momentum of the positrons and the mass of the muon. Compare your result to the accepted value of

$$m_{0_\mu} c^2 = 106 \text{ MeV}. \tag{12}$$

Find the average projected length of the muon track, and from this calculate the muon momentum and the pion mass. Compare your result to the accepted value of

$$m_{0_\pi} c^2 = 140 \text{ MeV}. \tag{13}$$

10.2.3 Question

Why does the radius of curvature of each positron decrease along the track?

10.3 APPENDIX I

10.3.1 Momentum Measurements

If the momentum $p_\perp$ of the particle with charge $e = 1.602 \times 10^{-19}$ coulomb (C) is in a plane perpendicular to the applied magnetic field $\vec{\boldsymbol{B}}$ then the radius of curvature R of its path is given by an expression identical to the classical result:

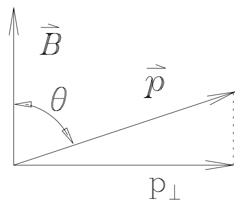
$$R = \frac{p_\perp}{eB} \tag{14}$$

The quantities in Eq. (14) must, of course, be expressed in a consistent set of units. Converting from mks (SI) or cgs units to a system where $p_\perp$ is expressed in MeV/c, it follows that

$$p_\perp\,[\mathrm{MeV}/c] \;= 3 \times 10^{-4}\; B\,[\mathrm{G}]\; R\,[\mathrm{cm}] \tag{15}$$

where B is expressed in gauss (G) and R in cm. [*You are expected to prove this in your report.* Remember, the particles are *relativistic*.]

In the more general case where the angle between the initial momentum $\vec{\boldsymbol{p}}$ and the magnetic field direction $\vec{\boldsymbol{B}}$ is θ (rather than 90°), the particle travels in a helix:

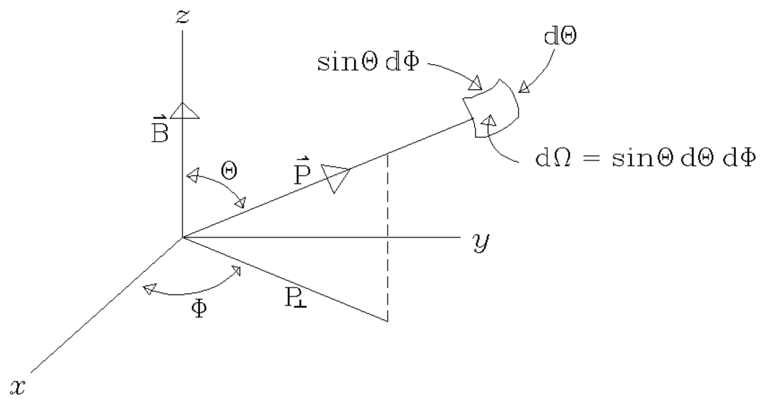


$$p = \frac{p_\perp}{\sin\theta}$$

In an actual research experiment, stereoscopic photography (using two, or sometimes three cameras), permits complete reconstruction of a track in space and thus gives a value of θ for each track. You will measure only one view in the present experiment, giving only the *projection* of the track on the $\theta = 90°$ plane. For a single track, you will be able to find only $p_\perp$, not p. If a large number of tracks are measured, however, the average momentum will be the average value of $p_\perp$ divided by the average value of $\sin\theta$:

$$\langle p \rangle = \frac{\langle p_\perp \rangle \quad [\text{measured}]}{\langle \sin\theta \rangle \quad [\text{calculated below}]} \tag{16}$$

Assuming that all directions of momenta are equally probable (as they are from the "isotropic" decay of a π meson at rest), the averaging of $\sin\theta$ is performed over area elements of a unit sphere; *i.e.* elements of solid angle $d\Omega$:



$$\langle \sin\theta \rangle = \frac{\int \sin\theta \, d\Omega}{\int d\Omega} \qquad \text{(Integrations are over the unit sphere)}$$

$$= \frac{\int_0^{2\pi} d\phi \int_0^{\pi} \sin^2\theta \, d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta} = \frac{\pi}{4} \simeq 0.785 \tag{17}$$

10.3.2 Energy Measurement for Stopped Particles

A charged particle loses energy in ionizing the material through which it passes. It can be shown that, to a good approximation, the energy (in MeV) lost per cm by a particle of charge e traversing liquid hydrogen in a bubble chamber is

$$\frac{dE}{dx} \, [\text{MeV/cm}] = \frac{2.1\rho \, \left[\text{gm/cm}^3\right]}{\beta^2} \tag{18}$$

where ρ is the density of the liquid in gm/cm^3 and $\beta \equiv v/c$. This energy loss is directly related to the number of bubbles per cm produced by the particle. Thus a particle traveling at almost the speed of light will produce a relatively sparse track, while a slow particle will produce a dense, almost solid track. This, combined with knowledge about the momentum p obtained from the radius of curvature, makes it easy to tell mesons from electrons and often mesons from protons. For example, a 10 MeV/c electron has a speed very close to c, while a 10 MeV/c meson has a speed about 0.1 c, so that its track would appear much denser.

Consider a particle which is created and also loses all its energy inside the bubble chamber so that its measured track length L corresponds to its range in liquid hydrogen. Then:

$$L = \int_0^T \frac{dx}{dE} dT \quad \text{or} \quad L \, [\text{cm}] = \int_0^T \frac{\beta^2}{2.1\rho} dT \tag{19}$$

from Eq. (18). For a non-relativistic particle,

$$T = \frac{1}{2}m_0 v^2 = \frac{1}{2}m_0 c^2 \beta^2 \tag{20}$$

Therefore

$$L = \frac{2}{2.1 m_0 c^2 \rho} \int_0^T T\, dT \tag{21}$$

giving

$$T = \sqrt{2.1 m_0 c^2 \rho L} \tag{22}$$

Again, the measured track length $L_\perp$ is related to the actual track length L by

$$L_\perp = L \sin\theta \tag{23}$$

If a large number of tracks are measured, L is related to the average of the measured track lengths $L_\perp$ by

$$L = \frac{\langle L_\perp \rangle}{0.785} \tag{24}$$

using the same argument that was used to derive Eq. (17).

10.4 APPENDIX II: Using the Program XMuon

Introduction

The bubble chamber slides have been converted to a format that can be viewed and analyzed with a computer. The program designed for this purpose is called XMuon, and it runs on a *Sun* workstation called UG. XMuon is used to view bubble chamber slides, determine the radius of curvature of positron tracks, and determine the length of muon tracks.

Using XMuon:

Your T.A. will log you into UG via an X-terminal. When the program starts, you will see a set of instructions and then the prompt `XMUON>`. Start the lab by typing the `start` command as follows:

```
XMUON>start
```

A list of all of the commands is available at any time by typing `help` at the prompt.

Scanning the bubble chamber photographs:

To view the next photo, type `next`; to view the previous one, type `prev`.

Calibrating the scale:

Each time you start to scan a new photo, you will need to calibrate the software so that it can convert screen pixels into distances (in cm) in the bubble chamber. Type `cal` and brief instructions will be given. There are four X scratchings in the photo, but only two of them are at the right distance for this calibration procedure, so make sure that you use the two shown in Fig. 10.1. If you have completed the calibration and are unhappy with it, or if you would like to recalibrate, just type `cal` again.

Measuring the radius of curvature (positrons) and track length (muons):

Once a photo has been calibrated and you have entered the point-picking mode (by typing `pick`), you may determine the radius of curvature for a given track by clicking on points on the track with the LEFT mouse button. You need at least three points, but 10–15 points are recommended to minimize random errors. Once you are finished selecting points, press the MIDDLE mouse button, and the program will tell you the radius of curvature. To repeat the process with another track, just type `pick` again.

You may measure the length of the muon track by typing `len`. Click on the endpoints of the muon track and the program will return the length of the track.[4]

[4]Note: the program assumes a *straight* muon track. Are the muon tracks really straight or are they curved? Estimate the effect of this assumption on your results.

10.5 APPENDIX III: Using the *Java* Applet

The bubble chamber slides have been converted to a format that can be viewed and analyzed over the Internet with any *Java*-capable Web browser (such as *Netscape 2.0* or higher) running on *any* computer platform. The *Java* applet is used to view bubble chamber slides, determine the radius of curvature of positron tracks and determine the length of muon tracks.

Invoking the *Java* Applet:

Use your Web browser to go to the URL ("location") http://cuphy3.phys.columbia.edu/laboratory/ and click on "Muon Applet" — after a short wait (while the *Java* code is downloaded to your computer) you should see "Meson Decay Java Applet" on your regular Web browser screen and a new window should open up with a bubble chamber image on the left and a set of controls on the right. If this does not happen, it may be that you do not have *Java* "turned on."[5]

There is a Help button in the controls that you can click on to get context-specific help with the applet.

What may not be immediately obvious is that the box just below the MODE label is a pull-down menu that you must use to select different "modes" (actions to perform). The applet starts up in Calibrate mode.

Calibrating the scale:

Each time you start to scan a new event, you will need to calibrate the software so that it can convert screen pixels into distances (in cm) in the bubble chamber. The applet reverts to Calibrate mode each time you load a new photo, for this reason. Click on Help for instructions if the process is not self-explanatory. There are four X scratchings in the photo, but only two of them are at the right distance for this calibration procedure, so make sure that you use the two shown in Fig. 10.1. If you have completed the calibration and are unhappy with it, or if you would like to recalibrate, just pull down the MODE menu and select Calibrate again or (simpler) just click on the Recalibrate button.

Measuring the track length (muons):

Once a photo has been calibrated you may measure the length of the muon track by selecting Measure Distance mode. Then just click on the endpoints of the muon track and the program will return the length of the track.[6]

[5]**Enabling *Java* in your Web Browser:** If you are using *Netscape 2.0* or higher, you should have *Java* capability built in. (If not, you may wish to download the latest version of *Netscape* for your computer platform from http://home.netscape.com/download/ where it is available free for educational use.) The *Java* capability must be "turned on" in order to work. This is usually the default, but if the applet fails to run for you, you may need to enable *Java* and *JavaScript* (see the Languages tab under Network Preferences in the Options menu in *Netscape 3.1*, for example).

[6]Note: the program assumes a *straight* muon track. Are the muon tracks really straight or are they curved? Estimate the effect of this assumption on your results.

Measuring the radius of curvature (positrons):

Once you have calibrated the photo and measured the length of a muon track, you may determine the radius of curvature for the positron track by selecting Measure Radius mode and clicking on points on the track. You need at least three points; 10–15 points are recommended to minimize random errors. Once you are finished selecting points, click on the Radius Done button and the program will draw a curve through the points and tell you its radius of curvature.

Selecting a new bubble chamber photograph:

There may be several $\pi - \mu - e$ decay "events" on any given photo. You should measure them all to avoid any bias in angle selection (recall the warnings in the **PROCEDURE** section). There is no need to keep track of which positron track is associated with which muon track.[7] To randomly select a new photo, click on the Next Image control button. To go to a specific photo (by number) click on the Goto Image # button.

[7]However, you might find it interesting to make note of whether there is any tendency for the positron to be emitted in the direction of motion of the muon (or opposite to it); there is an entertaining story behind this feature. . . .