Experiment 3

Simulations

Introduction

Each of the other experiments you perform in this laboratory involve a physical apparatus which you use to make measurements of various phenomena. This is the usual way in which physical laws such as Newton's Second Law, Hooke's Law and Snell's Law were discovered and verified. Other situations arise where we may have a very good understanding of many (if not all) of the laws governing some system, yet we may wish to determine how the application of these laws affects its behavior. Often a simulation offers a powerful method for such situations.

Rather than actually building an apparatus to measure its properties, we can program a computer to simulate the behavior of the system of interest. Obviously, the results of the computer are only as good as the accuracy of the simulation. Simulations are of great importance in physics; for example, they are widely used in particle physics to account for the interactions of particles with the measurement apparatus. They allow one, at little cost, to perform a wide range of experiments on many different systems. Combined with an actual experiment, simulations allow one to determine the behavior of the physical system through comparison of measured results to those from the simulation.

Simulation programs are often highly specialized, designed for a specific situation. In this experiment, we will be using a commercial program called *Interactive Physics* for the simulation of various mechanical systems. This program allows us to "build" different apparatuses and "perform experiments" on them. It is important to understand that this program isn't magic, it is only applying Newton's Laws to systems; all of the calculations it does are based on material you learned in Physics lecture courses. What the simulations do allow is the rapid computation of the dynamics and kinematics of systems of particles. It can perform many calculations (which would be quite tedious for us to do by hand) and therefore allows us to look at more complicated situations than we would calculate manually. It also allows us to vary the properties of the systems at will (without having to have new parts built *etc.*) and allows us to study the behavior of ideal systems (say without friction), or to introduce such phenomena as friction in a controlled way to study their effects. Of course, the cost of this flexibility is that we cannot discover new physical laws as we could with a real apparatus; laws which were not programmed into the simulation clearly won't be revealed in its use.

3.1 Simple Motion of an Object

Start the program *Interactive Physics* on the computer under Windows. Once it starts you will be presented with a screen with a menu bar across the top and a sub-window labeled `Untitled #1` which is where you will create your simulation. From the `File Menu` you can use the usual Windows commands, to open new simulations, open old ones, and save the present simulation. There is an extensive on-line help facility which you should use as you progress through the experiments.

Figure 3.1 Startup screen for *Interactive Physics*

3.1.1 Getting Started

The first simulation will be very simple. With the mouse, select the circle picture on the left-hand toolbar, then position a circle in the window. You position the center of the circle by clicking the mouse, which produces a standard-sized circle, along with 4 small squares. By grabbing one of the squares with the mouse you can resize the circle. Try to change the size of the circle.

Figure 3.2 A circle object created with the default size.

We are now ready to run the simulation. Click on `Run`, then observe what happens. You can stop the simulation by clicking `Stop`, then reset it by clicking `Reset`.

Rather than just watching the simulation, we can plot the position, velocity and acceleration of the object. First, select the object with the mouse by clicking on the object. You may need to select the `Pick` tool first by clicking on the arrow on the left hand toolbar. Next, pull down the `Measure` toolbar item, and select either `Position, Velocity, Acceleration`, or all three with `P-V-A`. You are given the choice of measuring either the `x` or `y` components of each of the quantities, or both (except for in the `P-V-A` graph). Try running the simulation and measuring various kinematic variables of the mass. You can change the graph appearance by clicking on the arrow on the top left corner of the measurement window. Try each of the different graph types.

Figure 3.3 Measurement submenus.

You can also vary the properties of the circle you are studying. Select the circle by clicking on it, then type `Alt-Enter` (at the same time) which should present you with a new window entitled `Properties`. You can select any of the objects, forces or constraints which are defined in your simulation with the scrolling window or by clicking on that object. Available properties include the mass, velocity, moment of inertia, charge *etc.* You can also change the initial velocity of the object by clicking at the center of the object, then dragging the mouse in the direction you wish for the velocity vector. The length of the arrow which appears is proportional to the velocity. Vary the initial velocity of the object and observe the effects on its motion.

We can introduce several objects and allow them to interact. To illustrate this, open a new window (from the `File` menu item) and create a circle object with no initial velocity. Then create a poly-line or rectangular object by selecting the item on the left-hand toolbar. Draw an object underneath your circle. If you run the simulation at this point, both objects fall under gravity and never interact. We wish the fix the rectangular object: we do this by attaching an anchor to it. Select the anchor picture from the left hand toolbar and then attach it to the rectangle by placing the pointer over the object and clicking. Now try running the simulation. What happens? The simulation is designed to simulate "real" materials, and therefore includes effects of elasticity (the co-efficient of restitution) and friction. By selecting an object in the `Properties` window you can change the elasticity or friction coefficient. When a property actually depends on two objects, such as the elasticity coefficient, the simulation uses the smaller of the two for the collision. For example, if both objects have an elasticity of 0.5 the simulation will give the same results as if one were 0.5 and the other were 1.0 or 0.8. Set the elasticity coefficient of the rectangular surface to 1.0, then run the simulation for several values of the circle's elasticity between 0.5 and 0.99.

Qualitatively, what happens as you vary the elasticity?

Although you can see what is happening in the simulation while you vary parameters, it is difficult to get a clear quantitative result. We can export the results of any simulation for any variable we can measure (using a Meter in the terminology of the program) and the `Export Data` function of the `File` menu. To use this feature, set up the simulation you wish to run and run it. Then, `Stop` and `Reset`. Select the `Export Data` function and enter a filename when prompted. Use something like your last name and a number for the filename (*e.g.* `Smith1`) and take the default extension of `.dta` so that the files can be deleted later when the experiment is complete. When you complete the name and select the checked box, the results of the simulation will be written to the selected file as the simulation runs. You can then read this file with another program such as an editor (`Notepad` for example).

Run the simulation for a series of elasticity values between roughly 0.5 and 0.99, saving the results in a series of files. Be sure to keep track of which files contained which data as you go along (or you will end up with many unlabelled files). Do this for perhaps 5 different values of elasticity. Edit the files using the Windows Notepad editor. Draw a graph of the height reached after each bounce vs. the the number of the bounce. You can use the same graph for each of the different trials, but use different symbols for different values of elasticity.

For each file, calculate the ratio between adjacent maximum heights following each bounce. Does the maximum height decrease by a similar amount for a given value of elasticity? Calculate the average decrease in the maximum for each elasticity as well as the standard deviation in the mean (as a measure of its uncertainty) and graph it as a function of the elasticity. Note that the maximum height is proportional to the gravitational potential energy and therefore the total mechanical energy in the system. From your graph, how does the rate of mechanical energy loss depend on the elasticity of the collision?

3.2 Collisions in Two Dimensions

Open a new simulation window. For this simulation we will turn off gravity, so that we can simulate collisions on a frictionless surface. Under the `World` Menu, select `Gravity`, then click on `None`. Create two identical circles. You can change the properties of the individual circles by selecting one of them, typing `Alt-Enter`, then changing the desired property, such as location, velocity, density, elasticity *etc.* Set the positions of the two circles so that one can travel with its initial velocity along the x-axis and collide with the second. Arrange the relative positions so that the two circles don't have a straight-on collision, but rather, hit at a glancing angle.

Figure 3.4 Collisions in 2-d.

For the first series of collisions, set the elasticity coefficients for the two circles to 1 and set the frictional coefficients to zero. Collide the two objects and observe what happens. Measure the angle between the paths of the two recoiling circles. This can be done most conveniently by measuring the velocities of the recoiling circles. What do you find? Vary the degree to which the objects hit collide at glancing angles and measure the angle between the recoiling circles for 5 different collisions. Be sure to record the initial positions and velocities of the colliding objects.

Pick a convenient starting condition for the next series of collisions. Vary the elasticity coefficient of one of the circles and measure the collisions for 5 different elasticities. You should measure the angle between the recoiling balls, their speeds and the change in the kinetic energy during the collision.

For the next set of collisions vary the frictional coefficients (keep static and kinetic coefficients equal to each other for simplicity). Measure collisions with five different frictional coefficients; repeat this for two different values of the elasticities (make one of them 1.0). Calculate the total change in the translational and rotational kinetic energies during the collisions. What do you find for the angle between the paths of the recoiling circles for these collisions?

For the final set of collisions change the mass of the target circle. Choose the same starting conditions as you used above, and set the frictional coefficients to zero and choose elasticities of 1.0. Be sure to change the mass of the target by changing its density, not its size. Measure the angle between the recoiling circles for 5 different masses of the target ball, varying it from less than to more than that of the initially moving circle.

Comment on your results.

3.3 Damped Harmonic Motion

In this section of the experiment, you should design an experiment to measure simple harmonic motion. There are (at least) three different types of system you can simulate using *Interactive Physics*.

A. Mass on a Spring.

B. Simple or Physical Pendulum.

C. Torsional Oscillator.

3.3.1 Mass on a Spring

We will illustrate damped harmonic motion with a mass on a spring; however, you can choose either of the other systems mentioned above for your experiment.

Consider a mass m on a spring with spring constant k, as shown in Fig. 3.5. In addition to the spring force acting on the mass ($-kx$), we will also assume that dissipative forces act on the mass. These dissipative forces could be generated within the spring itself, or due to the fluid through which the mass moves. We will assume that the dissipative forces can be written as $F_{\text{dissipative}} = -bv$ where v is the velocity of the mass.

Figure 3.5 Mass on a spring

We can write $F = ma$ for the mass:

$$m\frac{d^2x}{dt^2} = -kx - bv \tag{1}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 \tag{2}$$

This is a second order differential equation whose general solution should have two functions with two arbitrary constants. We can solve by rewriting Eq. (2) as follows:

$$\left[\left(\frac{d}{dt} + \frac{b}{2m}\right) + \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right]\left[\left(\frac{d}{dt} + \frac{b}{2m}\right) - \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right] x = 0 \tag{3}$$

where the operator $\frac{d}{dt}$ acts on all terms to the right of it. The general solution of Eq. (3) is the sum of solutions obtained from setting the result of each of each first order operation on x equal to zero.

$$\left[\frac{d}{dt} + \frac{b}{2m} + \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right] x = 0 \tag{4}$$

$$\left[\frac{d}{dt} + \frac{b}{2m} - \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right] x = 0 \tag{5}$$

We can set $\gamma = \frac{b}{2m}$ and $q = \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}$ giving:

$$\left[\frac{d}{dt} + \gamma + q\right] x = 0 \tag{6}$$

$$\left[\frac{d}{dt} + \gamma - q\right] x = 0 \tag{7}$$

which gives for $x(t)$:

$$x(t) = A_1 e^{-(\gamma-q)t} + A_2 e^{-(\gamma+q)t} \tag{8}$$

Clearly q can be complex (for example, take $b = 0$, corresponding to the case where there is no damping). We can distinguish three different regimes for different possible values of q:

A. **Overdamped** (q real > 0)

Here $x(t)$ is a decaying function of time with two different decay time constants: $(\gamma - q)$ and $(\gamma + q)$. The amplitudes A_1 and A_2 are determined by the intial conditions (such as position and velocity).[1]

B. **Critically Damped** (q real $= 0$)

Here $q = 0$. Since the two first order equations are the same, they only give one function and constant. Since we need two independent functions and constants to satisfy possible boundary conditions we return to the original equation with $q = 0$.

$$\left(\frac{d}{dt} + \gamma\right)\left(\frac{d}{dt} + \gamma\right) x = 0 \tag{9}$$

We substitute

$$u = \left(\frac{d}{dt} + \gamma\right) x$$

giving

$$\begin{aligned}
\left(\frac{d}{dt} + \gamma\right) u &= 0 && (10)\\
u &= Ae^{-\gamma t} = \left(\frac{d}{dt} + \gamma\right) x && (11)\\
A &= e^{\gamma t}\left(\frac{d}{dt} + \gamma\right) x && (12)\\
&= \frac{d}{dt}\left[xe^{\gamma t}\right] && (13)
\end{aligned}$$

Integrating both sides with respect to time gives

$$\begin{aligned}
At + B &= xe^{\gamma t} && (14)\\
x(t) &= Ate^{-\gamma t} + Be^{-\gamma t} && (15)
\end{aligned}$$

[1] Equation (8), along with the definitions of γ and q, provides a surprisingly rich phenomenology which is worth playing with to gain familiarity; the simulation software offers a nice opportunity to "see" the behavior predicted by the equations. For example, consider what happens as $k \to 0$ ($q \to \gamma$): one of the terms in Eq. (8) becomes time-independent, expressing the result that an object moving under *only* a damping force may come to rest at some arbitrary position, not necessarily at $x = 0$. By contrast, under a linear restoring force (no matter how small) the damped motion must always approach $x = 0$ asymptotically. You might also wish to check that the constraint $dx/dt = 0$ at $t = 0$ (object initially at rest) imposes a constraint on the *ratio* of the amplitudes, $A_1/A_2 = (q + \gamma)/(q - \gamma)$.

Spend some time exploring these equations.

C. Underdamped (q imaginary)

With q imaginary, we can write $q = i\omega_d$ where

$$\omega_d = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\omega_0^2 - \gamma^2}$$

where ω_0 would be the oscillation frequency of the system in the absence of damping.

We can rewrite the general solution [Eq. (8)] for $x(t)$:

$$\begin{aligned} x(t) &= c_+ e^{-(\gamma - i\omega_d)t} + c_- e^{-(\gamma + i\omega_d)t} && (16) \\ &= e^{-\gamma t}\left[c_+ e^{i\omega_d t} + c_- e^{-i\omega_d t}\right] && (17) \end{aligned}$$

but x is real (after all it is the position of an object), therefore $x^* = x$ (x equals its complex conjugate). This gives:

$$x^* = e^{-\gamma t}\left[c_+^* e^{-i\omega_d t} + c_-^* e^{i\omega_d t}\right] = x$$

Therefore:

$$\begin{aligned} c_+^* &= c_- = c \\ c_-^* &= c_+ = c^* \\ \text{Therefore: } x(t) &= e^{-\gamma t}\left[c^* e^{i\omega_d t} + c e^{-i\omega_d t}\right] \end{aligned}$$

We can write

$$\begin{aligned} c &= \frac{A}{2} e^{-i\theta_0} \\ c^* &= \frac{A}{2} e^{i\theta_0} \end{aligned}$$

giving:

$$\begin{aligned} x(t) &= e^{-\gamma t}\left[\frac{A}{2}\right]\left[e^{i(\omega_d t + \theta_0)} + e^{-(\omega_d t + \theta_0)}\right] && (18) \\ &= A e^{-\gamma t}\cos(\omega_d t + \theta_0) && (19) \end{aligned}$$

Experiment

The same equations for damped harmonic motion apply for any of the types of damped harmonic motion described above. Here, choose either the mass on a spring or the torsional oscillator.

Open a new simulation and place a mass in it. Select the spring icon and connect the spring to the mass. You can select either the rotational spring or a linear spring. Run the simulation. Measure the position (translational or rotational) of the mass as a function of time. Determine the frequency of the motion by measuring the time for a number of complete oscillations. Compare the measured frequency to the theoretical value.

Now add damping to your simulation by adding a dashpot (also from the spring menu). If you are using the torsional spring, use a torsional dashpot. Choose different values of the damping strength (which you can change numerically through the `Properties` menu and choosing the constraint corresponding to the dashpot). Measure and graph the position (translational or rotational as appropriate) of the mass for the cases of underdamped, critically damped and overdamped motion.

Comment on your results.