Experiment 9

Oscilloscope and AC Circuits

The Oscilloscope

The cathode ray oscilloscope is a useful tool in experimental physics and electronics. It can be used to measure potential differences which change rapidly with time – too rapidly to be followed by a simple voltmeter. It is important that you become familiar with the use of the oscilloscope; you are encouraged to explore its operation by manipulating all of its controls.

There is one precaution which you must take in order to protect the instrument: **When the spot on the screen is stationary keep the intensity very low,** in order not to damage the fluorescent screen by "burning a hole" in it at that point. In general, do not leave the intensity higher than necessary for reasonable visibility.

The Cathode Ray Tube

Figure 9.1 shows schematically the essential features of the cathode ray tube in the oscilloscope.

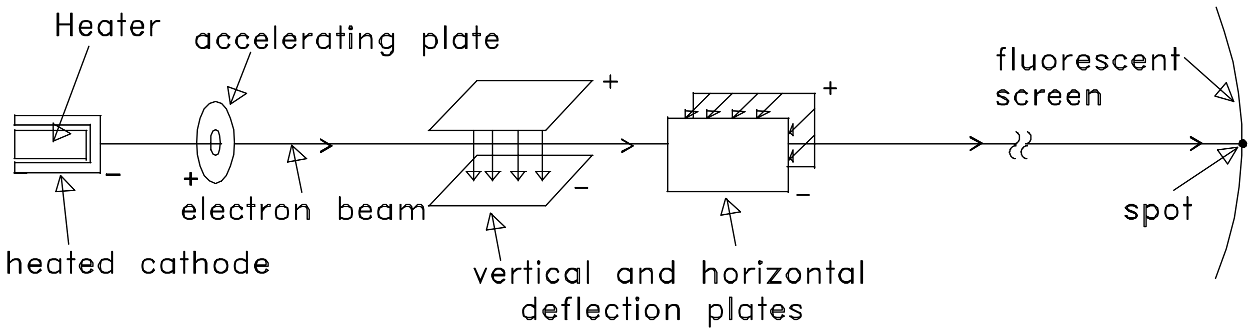

Figure 9.1 – The cathode ray tube (schematic)

Electrons leave the heated cathode by thermionic emission. They are accelerated through a fixed voltage and emerge as a narrow beam focussed through a hole in the accelerating plate. When the electron beam strikes the fluorescent screen on the face of the tube, it produces a small luminous spot. An external potential difference can be measured by applying it across a pair of parallel deflecting plates, through which the beam passes on its way to the screen. The beam is then deflected by the resultant transverse uniform electric field between the plates. There are two pairs of deflecting plates, one for vertical and the other for horizontal deflection.

If you double the externally applied potential across a pair of deflecting plates, you will also double

A. the electric field intensity between the plates.

B. the transverse force on the electrons.

C. the resultant transverse acceleration of the electrons, and thus,

D. the net transverse displacement and tangent of the angle at which the beam leaves the region between the plates.

The net displacement of the spot on the screen will therefore be proportional to the applied potential difference. It is this property which makes the oscilloscope useful as a voltmeter.

Display of Time-Varying Potential Difference

In order to display the time variation of a potential difference which is applied across the vertical deflection plates, the beam can be deflected horizontally by an internally-generated voltage which increases linearly with time. Figure 9.2 indicates the "sawtooth" voltage which, when applied to the horizontal deflection plates, sweeps the beam horizontally across the screen at constant velocity, returns the beam to its initial position and repeats the sweep, *etc.*

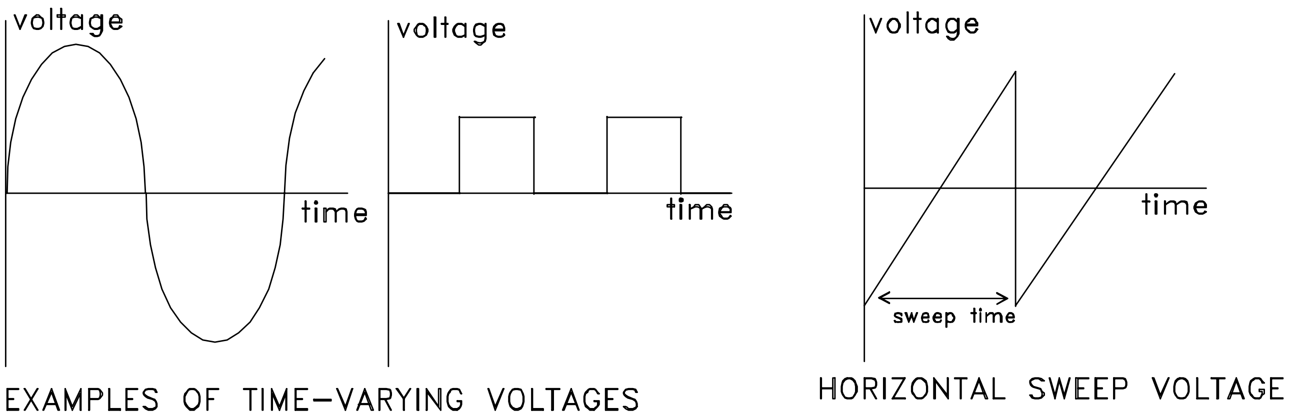

Figure 9.2 – Horizontal sweep voltage

The process is somewhat similar to writing on paper. To write, one "wiggles" the pen up and down while moving the hand horizontally across the paper. Here, the time-varying voltages applied to the vertical plates "wiggles" the electron beam up and down while the linear sweep voltage moves the beam horizontally with constant velocity. The beam leaves its written trace on the fluorescent tube screen. Because of the persistence of vision (approximately 0.05 s) and because of the screen fluorescence, we see a plot of the potential as a function of time displayed on the screen.

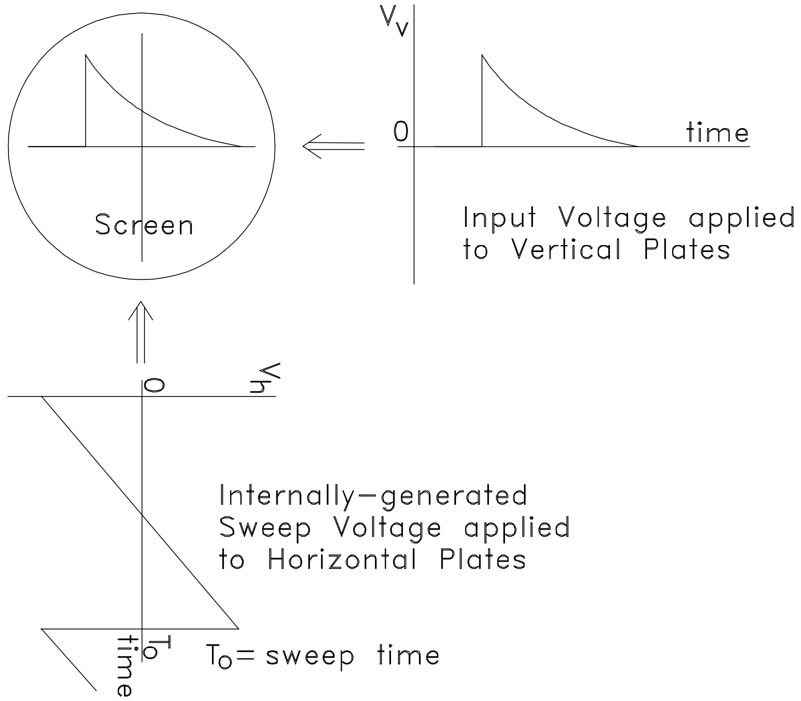

Figure 9.3 – The image on the screen is formed by combining the constant horizontal sweep with a vertical deflection proportional to some time-dependent input voltage which you want to observe.

Usually one observes the repeating pattern of a periodically varying voltage by appropriate adjustment of the sweep time and synchronization of the start of successive sweeps. Oscilloscope controls allow one to determine when the sweep initiates (in other words, when the sweep is "triggered"). See the description of the horizontal sweep trigger controls in the oscilloscope instructions which follow.

Use of Oscilloscope

A. Familiarize yourself with the operation of the oscilloscope by trying each of the controls. Apply a DC input voltage from the power supply. Then substitute an AC input voltage from the signal generator. Observe the effect of different sweep rates as well as the effect of applying a voltage to the horizontal input.

B. Use the signal generator to put sine waves and square waves of various frequencies on the vertical plates. Vary the amplitude of the input voltage and measure:

(a) The wave amplitude in volts: when the variable voltage sensitivity knob (10) is turned completely clockwise and the vertical sensitivity magnifier (11-B) is pushed in, the numbers on the vertical sensitivity VOLTS/DIV (15) control give the volts/division of the vertical grid. Using these, one can convert the centimeters (divisions) that one measures vertically on the screen into volts.

(b) The wave period in seconds: when the variable sweep time knob (22) is turned completely clockwise and the sweep time magnifier (24-B) is pushed in, the numbers on the sweep time TIME/DIV (23) control give the time for the beam to sweep horizontally one division. Knowing this one can convert the centimeters (divisions) that are measured horizontally into seconds.

RC Transients

When a voltage V_0 is applied to a series R-C circuit, the charge q across C rises asymptotically to Q_0 and the current i decays from its initial value towards zero. The rise of q and the fall of i both involve the same exponential function of time.

$$q(t) = Q_0 \left[1 - e^{-\frac{t}{RC}}\right], \quad i(t) = i_0 e^{-\frac{t}{RC}} \tag{1}$$

The value $\tau = RC$ is called the "time constant" of the circuit.

Connect the square wave generator, which alternatively places a fixed voltage and a short circuit across a series RC circuit for equal time intervals, as shown in Fig. 9.4. The rise and decay of i can then be observed by using the oscilloscope to measure the voltage across R as a function of time. [The oscilloscope measures $V(t)$; therefore $i(t) = V(t)/R$.] Be sure to connect your circuit so that the square wave generator and the oscillator share a common ground.

Adjust the period of the square wave generator to allow several time constants between consecutive "switching" and adjust the sweep time of the oscilloscope to observe the rise and decay of the current.

Measure $\tau = RC$ by determining the time it takes the current to fall to $1/e$ of its initial value.

The variation of the charge on the capacitor may be observed by using the oscilloscope to measure the voltage V_c across C:

$$V_c(t) = \frac{q(T)}{C} \tag{2}$$

Reconnect your circuit, again making sure that the square wave generator and the oscilloscope share a common ground.

A. Sketch graphs of $q(t)$ and $i(t)$, using a common time (horizontal) axis.

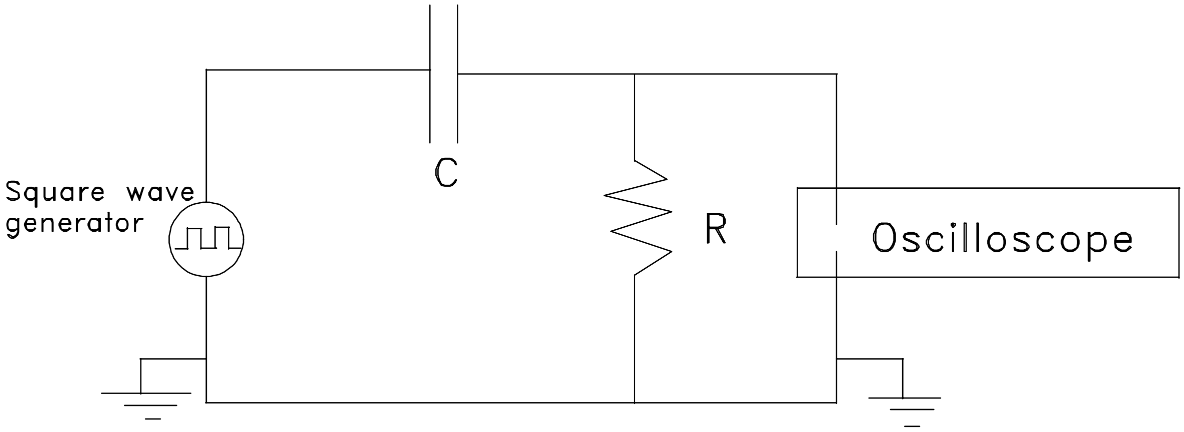

Figure 9.4 – Square wave generator to measure RC time constant.

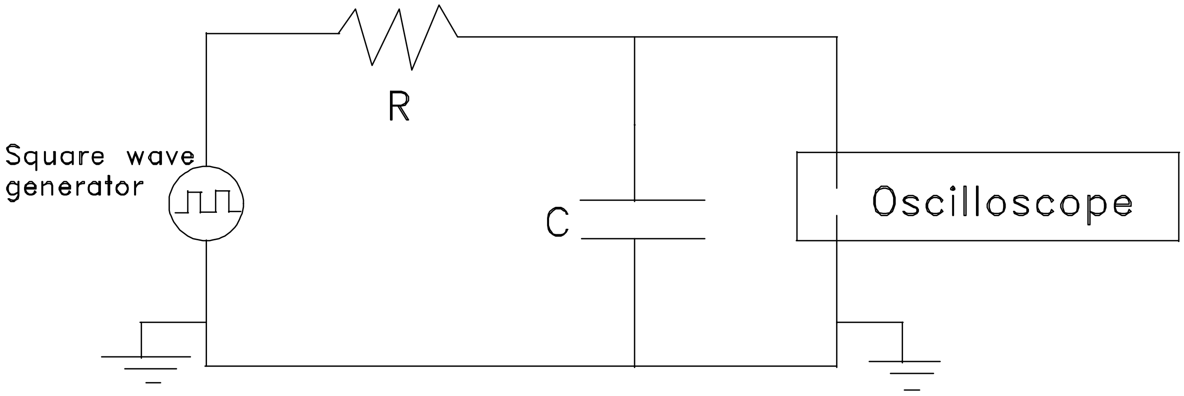

Figure 9.5 – Measuring V_c.

B. Compare your measured τ with the expected values from the given values of R and C.

C. What is the shortest value of τ you could conveniently measure with this technique? Explain briefly.

The value of τ will depend on the internal resistance of the square wave generator as well as the external resistor used: $\tau = (R + R_{\rm int})C$. The internal resistance $R_{\rm int}$ can be measured in the following way:

A. Connect the oscilloscope directly across the generator and measure the voltage V_1.

B. Connect the oscilloscope across the generator when it is connected in parallel with a resistor r and measure the voltage V_2.

C. Adjust r until $V_2 = V_1/2$. Then, $r = R_{\rm int}$. Explain why this is true.

AC Circuits

We will restrict our attention in the following to the long-time behaviour of AC circuits; in particular, we will examine the response of circuits at the same frequency as the driving source, neglecting any transient responses.

The voltage drop across a resistor (R), capacitor (C) and inductor (L) are given as:

$$\begin{aligned} V_R &= IR & (3) \\ V_C &= \frac{Q}{C} = \frac{\int I dt}{C} & (4) \\ V_L &= L\frac{dI}{dt} & (5) \end{aligned}$$

We can represent the current by a complex exponential:

$$I = I_m e^{i\omega t} \tag{6}$$

If we now evaluate V_R, V_C and V_L:

$$\begin{aligned} V_R &= I_m e^{i\omega t} R = IR & (7) \\ V_C &= \frac{1}{C}\int I_m e^{i\omega t} dt & (8) \\ &= \frac{1}{i\omega C} I_m e^{i\omega t} & (9) \\ &= \frac{-i}{\omega C} I & (10) \\ V_L &= L\frac{dI}{dt} & (11) \\ &= L I_m (i\omega) e^{i\omega t} & (12) \\ &= i\omega L I & (13) \end{aligned}$$

We can define the *complex impedance* X:

$$\begin{aligned} X_R &= R & (14) \\ X_C &= \frac{-i}{\omega C} & (15) \\ X_L &= i\omega L & (16) \end{aligned}$$

so that the voltage drop across any element is given simply as

$$V = XI. \tag{17}$$

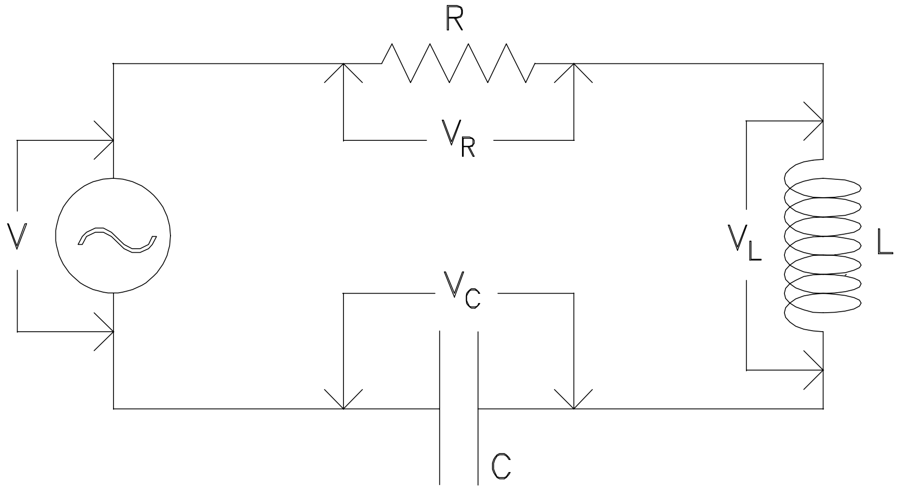

Figure 9.6 – A driven series RLC circuit.

Consider the circuit in Figure 9.6 consisting of a resistance R, a capacitance C and an inductance L, connected in series and driven by a time-varying voltage source.

The differential equation for the current flow is obtained by equating the voltage of the source to the total potential drop around the circuit.

$$V = V_R + V_L + V_C. \tag{18}$$

We write the driving sinusoidal voltage as $V = V_m e^{i\omega t}$, then take the current to have the same frequency, but a relative phase ϕ:

$$I = I_m e^{i(\omega t - \phi)}. \tag{19}$$

To determine the response of the circuit, we need to determine I_m, ϕ.

$$\begin{aligned}
V_m e^{i\omega t} &= IR + IX_C + IX_L && (20)\\
&= I_m e^{i(\omega t-\phi)} R + \frac{-i}{\omega C} I_m e^{i(\omega t-\phi)} + i\omega L I_m e^{i(\omega t-\phi)} && (21)\\
V_m &= I_m \left[R e^{-i\phi} - \frac{i}{\omega C} e^{-i\phi} + i\omega L e^{-i\phi} \right] && (22)
\end{aligned}$$

Since the left hand side of the equation for V_m is purely real, the right hand side must also be real; in other words, the imaginary part must equal zero. Making use of the identity $e^{i\omega t} = \cos\omega t + i \sin\omega t$, we obtain:

$$\begin{aligned}
0 &= R \sin -\phi - \frac{\cos\phi}{\omega C} + \omega L \cos\phi && (23)\\
\tan\phi &= \frac{\omega L - \frac{1}{\omega C}}{R} && (24)
\end{aligned}$$

From our result for $\tan\phi$, we can construct a triangle and use the Pythagorean theorem to obtain

$$\sin\phi = \frac{\omega L - \frac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{25}$$

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{26}$$

Substituting into the real part of Eq. (22) we obtain:

$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}. \tag{27}$$

Frequency Dependence of Impedance

We wish to measure the magnitudes of the impedances X_C and X_L by measuring V_C and V_L which will be proportional to the impedances if I_m is kept constant. However, as the frequency of the signal generator is varied, not only does the impedance of L and C change, but the impedance of the whole circuit changes. Hence, if V_m (the amplitude of the signal generator's output) is left constant when the frequency is changed, I_m will therefore change. To keep I_m constant, we need to adjust V_m so that the voltage across the resistor R remains constant.

Use the oscilloscope to measure the voltage drop first across L and then across C for a wide range of frequencies, measuring V_R between each change of frequency and adjusting V_m to keep V_R (and therefore I_m) constant.

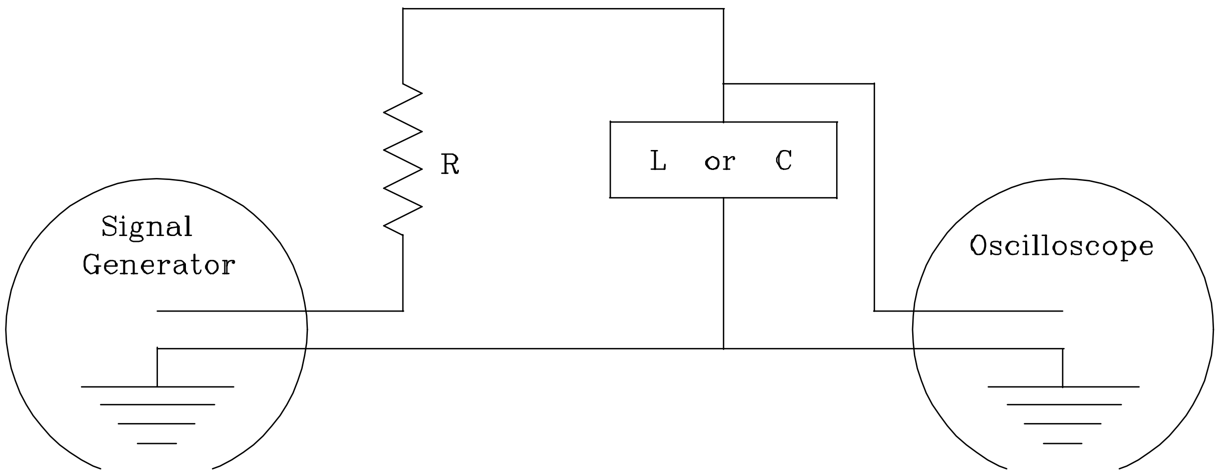

Figure 9.7 – Measuring V_L or V_C.

To measure V_R, the circuit must be reconnected so that the signal generator and the oscilloscope share a common ground.

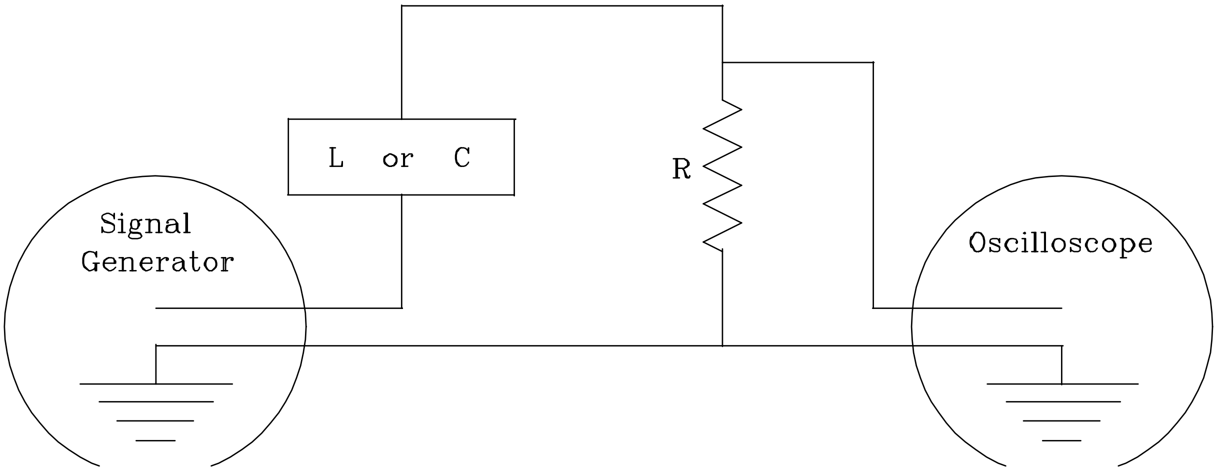

Figure 9.8 – Measuring $V_R = RI_m$.

You should check the frequency settings of the signal generator by measuring the periods of oscillation with the oscilloscope.

Plot the results (choosing variables for ordinate and abscissa with some forethought) so as to make a simple graphical comparison with the equations for X_C and X_L. Note that if $y = kx$, then the graph of y vs. x should be a straight line. If $y = k/x$, then the graph of y vs. $1/x$ should be a straight line. What values of L and C are implied by your graphs?

In practice, the elements may not be pure R, L and C. The resistor may have some associated inductance, the inductance may have an appreciable resistance, *etc.* Do your curves show any evidence of these effects (do they show any systematic deviations from the expected behaviour)? Is there any danger that connecting the oscilloscope will alter the behaviour of the circuit (the 'scope has an input resistance of 1 MΩ, with an input capacitance, in parallel, of 35 pF)?

Voltages in the Series RLC Circuit: Magnitudes and Phase Shifts

In this part of the experiment, you will connect R, L and C in series with a signal generator and use the oscilloscope to measure the magnitude of the voltage across each circuit element and the relative phase of each voltage with respect to the voltage of the signal generator.

The oscilloscope and the signal generator must share a common ground, so you will have to rearrange your circuit each time you measure the voltage across a different circuit element. For example, to measure V_C the arrangement could be as shown in Fig. 9.9: but then to measure V_L, the inductor would need to be attached to the ground of the signal generator as shown in Fig. 9.10:

A. Measure the magnitudes of V_R, V_L and V_C.

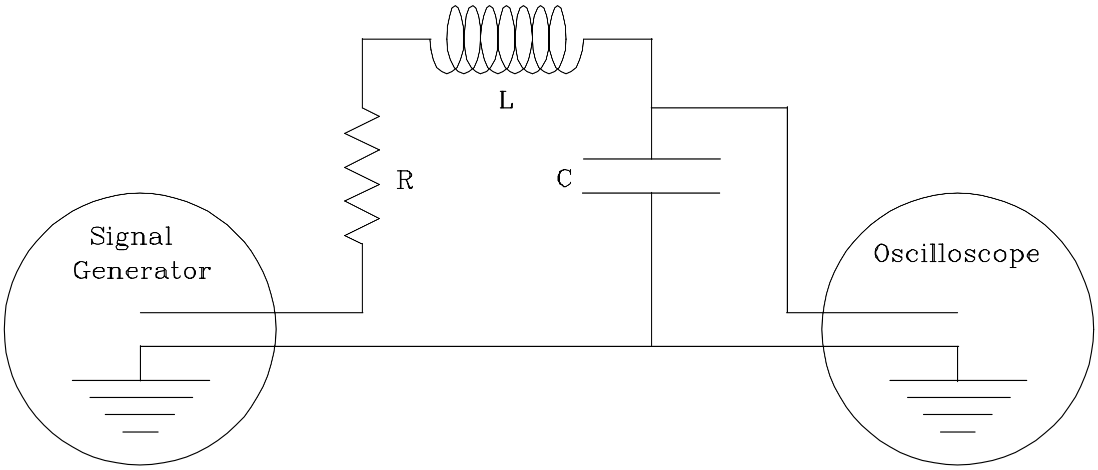

Figure 9.9 – Measuring V_C.

B. Measure the phase shifts between these voltages and the the voltage V of the signal generator. For the phase measurements, the signal generator is used as an **external trigger** to the oscilloscope. In this mode (SOURCE button pushed to EXT), it is the phase of of the input to the external trigger rather than the phase of the vertical input that triggers the start of the sweep, so that the relative phases of the two inputs can be compared.

For measuring phase shifts,

(*a*) First center the pattern vertically by pushing the ground button in and adjusting the resulting line to coincide with the center line of the grid. Then, let the ground button out.

(*b*) With the output from the signal generator as the vertical input to the oscilloscope (as well as the external trigger input), adjust the time/div selection switch and the variable horizontal scale knob so that one cycle (or one-half cycle) covers nine centimeters – a convenient scale for measuring phase shifts as each centimeter represents 40 (or 20) degrees. Position the trace horizontally so that one (or two) of the center line crossings coincides with a vertical grid line – your reference line for the next step.

(*c*) Use V_R, V_L or V_C as the vertical input and keep the signal generator as the external trigger input. The horizontal change in position of the center line crossing is the phase shift you wish to measure. Sketch the trace in your lab notes so you can remember the direction of the phase shift. Remember that a positive shift moves the pattern to the left.

C. From the measured phase shifts, determine the phase shifts between V_L and V_R and between V_C and V_R. How do your values compare with what you expect?

D. Assuming that the value of R is known, determine (from R, the voltages and the frequency) values of C and L. Compare the phase shift of the current relative to the driving voltage that you calculate from R, L, C with the measured value.

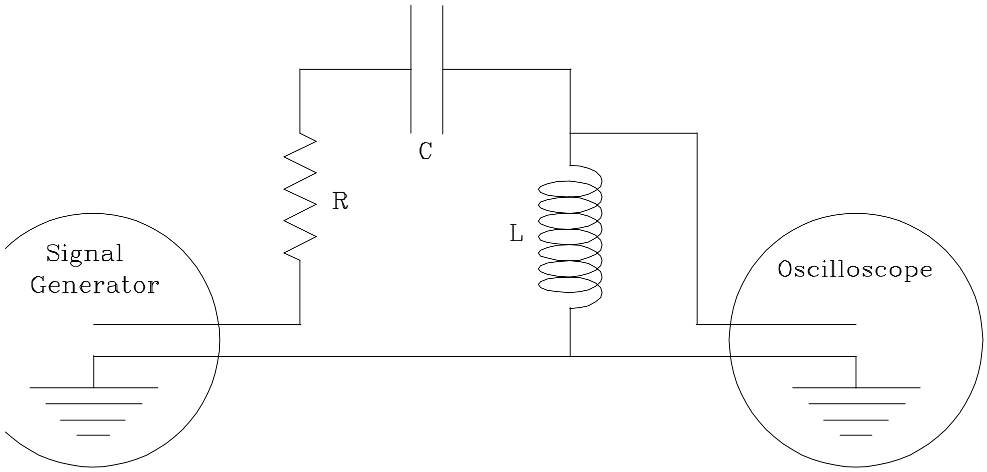

Figure 9.10 – Measuring V_L.

E. Compare your experimental results with the values of L and C labelled on the devices.

Resonance in a Series RLC Circuit

The series RLC circuit has a resonance where the current takes a maximum value. This resonance will be sharpest for small values of R, large L and small C, although the frequency of the resonance depends only on L and C. Suggested values are $L \approx 25$ mH and $C = 0.1$ μF, since with these values the resonant frequency is not near the end of a decade on the signal generator. Any value of R between 10 and 100 ohms will work well – most of the resistance in your circuit is the internal resistance of the signal generator. Observe how the voltage across the resistor (and hence the current) changes as the frequency is changed. Determine the value of ω for which the circuit's total impedance is minimized and compare this with what you would expect given the values of L and C in your circuit.

Measure the current I_m in the circuit as a function of frequency in the vicinity of the resonance. Graph I_m *vs.* ω sweeping through the resonance.